MAT502: MODULE THEORY

UNIT I: Modules over a ring, Endomorphism ring of an abelian group, R-Module structure on an abelian group M as a ring homomorphism from R to $End_Z(M)$, submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Statements of Correspondence theorem and Isomorphism theorems, $Hom_R(M, N)$ as an abelian group and $Hom_R(M, M)$ as a ring, Exact sequences, Five lemma, External and internal direct sums and their universal property.

UNIT II: Free modules, Homomorphism extension property, equivalent characterization as a direct sum of copies of the underlying ring, existence of a basis of a vector space, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules.

UNIT III: Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Divisor chain condition, Unique factorization domains, Examples and counterexamples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Unique factorization in polynomial rings over UFD's.

UNIT IV: Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into $T(M)$ and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

~~**UNIT V:** Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$-module structure, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Diagonalizable and nilpotent parts of a linear operator, Jordon-Chevalley Theorem~~

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, N.Y., 1974.
3. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. P. Ribenboim, Rings and Modules, Wiley Interscience, N.Y., 1969.
3. J. Lambek, Lectures on Rings and Modules, Blaisedell, Waltham, 1966.
4. Ramji Lal, Algebra, Vols. II, Shail Publications, Allahabad, 2002.

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MAT504: MEASURE AND INTEGRATION

UNIT I: Countable and uncountable sets, cardinality and cardinal arithmetic, Schröder-Bernstein theorem, $a < 2^a, 2^{\aleph_0} = c$, the Cantor's ternary set and its properties.

UNIT II: Semi-algebras, algebras, monotone class, σ-algebras, measure and outer measures, Caratheödory extension process of extending a measure on a semi-algebra to generated σ - algebra, completion of a measure space.

UNIT III: Borel sets, Lebesgue outer measure and Lebesgue measure on $\mathbb{R}$, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets, the Cantor-Lebesgue function.

UNIT IV: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, simple functions and their integrals on $\mathbb{R}$, Littlewood's three principles (statement only), Lebesgue integral on R and its properties.

UNIT V: Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, Minkowski's and Hölder's inequalities, Riesz-Fischer theorem (statement only).

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Books Recommended:

H. L. Royden and P. M. Fitzpatrick, Real Analysis, (Fourth Edition), Prentice Hall, 2010.

Further Reading:

1. P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
2. E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.
3. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM 33, Hindustan Book Agency, New Delhi, 2005.
4. I. K. Rana, An Introduction to Measure and Integration, (Second Edition), Narosa Publishing House, New Delhi, 2005.

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MAT506: PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Unit-I: Formation of P.D.E's, First order P.D.E.'s, Classification of first order P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit-II: Pfaffian differential equations, Compatible systems, Charpit's method, Jacobi's Method.

Unit-III: Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.'s.

Unit-IV: Method of separation of variables:- Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations of strings and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

Unit-V: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books recommended:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill Pub.,(1957)
2. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Pub. (2005)
3. R.P. Kanwal: Linear Integral Equations, Birkhauser Verlag Pub.(1997)

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MAT 50[illegible]: MATHEMATICAL METHODS

UNIT I:

Boundary value problems, Orthogonal and Orthonormal sets of functions, Sturm-Liouville (S-L) problems. Eigenvalues and Eigenfunctions of S-L problems, Reality of Eigenvalues and Orthogonality of Eigenfunctions of S-L problems, Singular Sturm-Liouville problems. Mean square Convergence. Completeness of Orthonormal sets, Bessel's inequality, Self-adjoint differential equations. Orthogonal Eigenfunction Expansions, Generalized Fourier series.

UNIT II:

Fourier Series: Periodic functions, Trigonometric series, Fourier series, Euler formulas. A set of sufficient conditions for the convergence of Fourier series of a continuous function of period 2π. Functions of arbitrary periods, Even and Odd functions, Fourier Cosine and Sine series, Half-range expansions, Complex Fourierr series, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error, Bessel's inequality

UNIT III:

From Fourier Series to Fourier Integral, Sufficient conditions for the validity of Fourier integral representation. Fourier Cosine and Sine Integrals, Fourier Cosine and Sine Transforms, Linearity and Fourier Cosine and Sine Transforms of Derivatives, Complex form of Fourier Integral. Fourier Transform and its Inverse, Linearity , Shifting properties, Fourier Transform of Derivatives, Convolution

UNIT IV:

Definition, Linearity and existence of Laplace transform. The inversion formula, Laplace transform of the derivatives and of the integrals of a function, Unit step function Shifting Theorems, Derivatives and Integrals of Laplace Transforms, Convolution products. Application to the Initial Value Problems and System of ODE.

UNIT V:

Calculus of Variations: Functionals and extremals, Variation and its properties, Euler equations, Cases of several dependent and independent variables, Functionals dependent on higher derivatives, Simple applications.

Books Recommended:

1. 1.E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8thEdition, 2001.
2. 2.A. D. Polyanin and A. V. Manzhirov, Handbook of Integral Equations, CRC Press, 2ndEdition, 2008.
3. L. Elsgolts, Differential Equations and Calculus of Variations, Mir Publishers, 1970.
4. A. S. Gupta, Calculus of Variations, Prentice Hall of India, New Delhi, 1999.
5. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004
6. William E. Boyce and Richard C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley & Sons, (Asia), Seventh Edition, 2003.
7. Pipes,Applied Mathematics for Engineers and Physicists, McGraw-Hill International Student Edition, 2nd edition

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MAT 510 DIFFERENIIAL GEOMETRY II

Unit 1: n-dimensional real vector space, contravariant vectors, dual vector space, covariant vectors, tensor product, second order tensors, tensors of type (r, s), symmetry and skew symmetry of tensors, fundamental algebraic operations, quotient law of tensors.

Unit 2 : Topological manifolds, compatible charts, smooth manifolds, examples, smooth maps and diffeomorphisms, definition of a Lie group, examples.

Unit 3: Tangent and cotangent spaces to a manifold, derivative of a smooth map, *immersions* and submersions, submanifolds , vector fields, algebra of vector fields, φ-related vector fields, left and right invariant vector fields on Lie groups.

Unit 4: Integral curves of smooth vector fields, complete vector fields, flow of a vector field, distributions, tensor fields on manifolds, r-forms, exterior product, exterior differentiation, pull-back differential forms.

Unit 5: Affine connections (covariant differentiation) on a smooth manifold, torsion and curvature tensors of an affine connection, identities satisfied by curvature tensor.

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Books Recommended:

1. Kobayashi and Nomizu; Foundations of Differential geometry, Vol-1, Interscience Publishers, 1963.
2. T. J. Willmore; Riemannian geometry, Oxford Science Publication, 1993.
3. S. Kumaresan; A course in Differential Geometry and Lie groups, Hindustan Book Agency, 2002.
4. M. Spivak; A comprehensive Introduction to Differential Geometry, Vols. 1-5, Publish or Perish, Inc., Houston, 1999.
5. W. M. Boothby; An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.
6. U. C. De, A. A. Sheikh; Differential Geometry of Manifolds, Narosa Publishing House, 2007.
7. R. S. Mishra, A course in Tensors with Applications to Riemannian Geometry, Pothishala, Pvt. Ltd., Allahabad, 1965.