PAPER -2004

1. Which one of the following represents the correct dimensions of the coefficient of viscosity?
(A) $ML^{-1}T^{-2}$ (B) MLT^{-1}
(C) $ML^{-1}T^{-1}$ (D) $ML^{-2}T^{-2}$

1. C.
Dimensions of η (coefficient of viscosity)
$$= \frac{MLT^{-2}}{M^0L^0 \cdot M^0LT^{-1}} = ML^{-1}T^{-1}$$

2. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(A) x^2 (B) e^x
(C) x (D) $\log_e x$

2. A.
$$K_f - K_i = \frac{mk}{2}x^2$$
$$K_f - k_i \propto x^2.$$

3. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T/3 seconds?
(A) h/9 metres from the ground (B) 7h/9 metres from the ground
(C) 8h/9 metres from the ground (D) 17h/18 metres from the ground.

3. C.

4. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is
(A) π (B) $\pi/3$
(C) $\pi/2$ (D) $\pi/4$

4. A.

5. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two time of flights is directly proportional to
(A) $1/R^2$ (B) $1/R$
(C) R (D) R^2

5. C.
Range is same for complimentary angles.
$$T_1 = \frac{2u \sin\theta}{g} \text{ and } T_2 = \frac{2u \sin(90-\theta)}{g}$$
and $$R = \frac{u^2 \sin 2\theta}{g}$$
$$\therefore T_1T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2R}{g}.$$

6. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(A) The velocity vector is tangent to the circle.
(B) The acceleration vector is tangent to the circle.
(C) The acceleration vector points to the centre of the circle.
(D) The velocity and acceleration vectors are perpendicular to each other.

6. **B.**
The acceleration vector is along the radius of circle.

7. An automobile travelling with speed of 60 km/h, can brake to stop within a distance of 20 cm. If the car is going twice as fast, i.e 120 km/h, the stopping distance will be
(A) 20 m (B) 40 m
(C) 60 m (D) 80 m

7. **D.**
If the initial speed is doubled, the stopping distance becomes four times, i.e. 80 m.

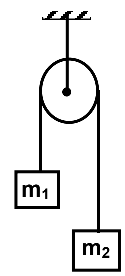
8. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(A) one (B) four
(C) two (D) three

8. **D.**
Change in momentum for each bullet fired is

$$= \frac{40}{1000} \times 1200 = 48 \text{ N}$$

If a bullet fired exerts a force of 48 N on man's hand so ρ man can exert maximum force of 144 N, number of bullets that can be fired = 144/48 = 3 bullets.

9. Two masses m_1 = 5 kg and m_2 = 4.8 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift free to move?
(g = 9.8 m/s^2)
(A) 0.2 m/s^2 (B) 9.8 m/s^2
(C) 5 m/s^2 (D) 4.8 m/s^2



9. **A.**

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = 0.2 \text{ m/s}^2$$

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(A) 7.2 J (B) 3.6 J
(C) 120 J (D) 1200 J

10. **B.**
Work done = mgh = 1.2 × 0.3 × 10 = 3.6 J.

11. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 m/s^2)
(A) 2.0 (B) 4.0
(C) 1.6 (D) 2.5

11. **A.**
m = 2 kg

12. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})m$. The work done on the particle in joules is
(A) –7 (B) +7
(C) +10 (D) +13

12. B.
Work done, W = $\vec{F} \cdot \vec{s}$
Here $\vec{s} = \vec{r}_f - \vec{r}_i = (2\hat{i} - \hat{j})$
$W = (5\hat{i} + 3\hat{j} + 2\hat{k})(2\hat{i} - \hat{j})$ = 10 – 3 = 7 J.

13. A body of mass m, accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is
(A) $\frac{mv_1 t}{t_1}$ (B) $\frac{mv_1^2 t}{t_1^2}$
(C) $\frac{mv_1 t^2}{t_1}$ (D) $\frac{mv_1^2 t}{t_1}$

13. B.
Power $P = \vec{F} \cdot \vec{v} = mav = m\left(\frac{v_1}{t_1}\right)\left(\frac{v_1}{t_1} t\right) = \frac{mv_1^2 t}{t_1^2}$

14. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that
(A) its velocity is constant (B) its acceleration is constant
(C) its kinetic energy is constant (D) it moves in a straight line.

14. C.
When a force of constant magnitude which is always perpendicular to the velocity of the particle acts on a particle, the work done and hence change in kinetic energy is zero.

15. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?
(A) moment of inertia (B) angular momentum
(C) angular velocity (D) rotational kinetic energy.

15. B.
Let it be assume that in “free space” not only the acceleration due to gravity it acting but also there are no external torque acting but also there are no external torque acting on the sphere. If due to internal changes in the system, the radius has increased, then the law of the conservation of angular momentum holds good.

16. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(A) yes, 60° (B) yes, 30°
(C) no (D) yes, 45°

16. A.
For the person to be able to catch the ball, the horizontal component of the velocity of the ball should be same as the speed of the person.

$v_0 \cos\theta = \frac{v_0}{2}$

$\Rightarrow \theta = 60°$.

17. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that
(A) $I_A = I_B$ (B) $I_A > I_B$
(C) $I_A < I_B$ (D) $I_A/I_B = d_A/d_B$
Where d_A and d_B are their densities.

17. C.
Moment of inertia of a uniform density solid sphere, A = $\frac{2}{5}MR^2$

And of hollow sphere B = $\frac{2}{3}MR^2$

Since M and R are same, $I_A < I_B$.

18. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is
(A) gx (B) $\frac{gR}{R-x}$
(C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

18. D.
For the satellite, the gravitational force provides the necessary centripetal force i.e.
$$\frac{GM_e m}{(R+X)^2} = \frac{Mv_0^2}{(R+X)} \text{ and } \frac{GM_e}{R^2} = g$$
$$\therefore\ v_0 = \left(\frac{gR^2}{R+X}\right)^{1/2}$$

19. The time period of an earth satellite in circular orbit is independent of
(A) the mass of the satellite
(B) radius of its orbit
(C) both the mass and radius of the orbit
(D) neither the mass of the satellite nor the radius of its orbit.

19. A.
The time period of satellite is given by
$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$
where, R + h = radius of orbit satellite, M = mass of earth.

20. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is
(A) 2 mgR (B) $\frac{1}{2}mgR$
(C) $\frac{1}{4}mgR$ (D) mgR

20. B.

21. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period planet in circular orbit of radius R around the sun will be proportional to

(A) $R^{\left(\frac{n+1}{2}\right)}$
(B) $R^{\left(\frac{n-1}{2}\right)}$
(C) R^n
(D) $R^{\left(\frac{n-2}{2}\right)}$

21. A.
$T \propto R^{(n+1)/2}$

22. A wire fixed at the upper end stretches by length □ by applying a force F. The work done in stretching is
(A) $F/2\ell$
(B) $F\ell$
(C) $2F\ell$
(D) $F\ell/2$

22. D.
Work done = $\frac{1}{2}kx^2 = \frac{1}{2}k\ell^2$ where ℓ is the total extensions.

$$= \frac{1}{2}(k\ell)\ell = \frac{1}{2}F\ell$$

23. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is
(A) directly proportional to R but inversely proportional to v.
(B) directly proportional to both radius R and velocity v.
(C) inversely proportional to both radius R and velocity v.
(D) inversely proportional to R but directly proportional to velocity v.

23. B.
Retarding viscous force = $6\pi\eta Rv$

24. If two soap bubbles of different radii are connected by a tube,
(A) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged.
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(C) air flows from the smaller bubble to the bigger.
(D) there is no flow of air.

24. C.
The pressure inside the smaller bubble will be more $\left(P_i = P_0 + \frac{4T}{r}\right)$

Therefore, if the bubbles are connected by a tube, the air will flow from smaller bubble to the bigger.

25. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\left(\frac{4}{3}\right) \times 1000\ \text{kg/m}^3$. What relationship between t and t_0 is true?
(A) $t = t_0$
(B) $t = t_0/2$
(C) $t = 2t_0$
(D) $t = 4t_0$

25. C.

$$\frac{T}{T_0} = \sqrt{\frac{1}{\left(1-\frac{\rho'}{\rho}\right)}} = \sqrt{\frac{1}{1-\frac{1}{3}}}$$

$$\Rightarrow \quad \frac{T}{T_0} = 2$$

or, $T = 2T_0$

26. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is t, then

(A) $T = t_1 + t_2$
(B) $T^2 = t_1^2 + t_2^2$
(C) $T^{-1} = t_1^{-1} + t_2^{-1}$
(D) $T^{-2} = t_1^{-2} + t_2^{-2}$

26. B.

$t_1^2 + t_2^2 = T^2$

27. The total energy of particle, executing simple harmonic motion is

(A) $\propto x$
(B) $\propto x^2$
(C) independent of x
(D) $\propto x^{1/2}$

27. C.

In simple harmonic motion, as a particle is displaced from its mean position, its kinetic energy is converted to potential energy and vice versa and total energy remains constant. The total energy of simple harmonic motion is independent of x.

28. The displacement y of a particle in a medium can be expressed as $y = 10^{-6}\sin(110t + 20x + \pi/4)$ m, where t is in seconds and x in meter. The speed of the wave is

(A) 2000 m/s
(B) 5 m/s
(C) 20 m/s
(D) 5π m/s.

28. B.

$$v = \frac{\omega}{k} = 5 \text{ ms}^{-1}$$

29. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to $\cos\omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to

(A) $\frac{m}{\omega_0^2 - \omega^2}$
(B) $\frac{1}{m(\omega_0^2 - \omega^2)}$
(C) $\frac{1}{m(\omega_0^2 + \omega^2)}$
(D) $\frac{m}{\omega_0^2 + \omega^2}$

29. B.

For forced oscillations, the displacement is given by

$$x = A\sin(\omega t + \phi) \text{ with } A = \frac{F_0/m}{\omega_0^2 - \omega^2}$$

30. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then
(A) $\omega_1 = \omega_2$
(B) $\omega_1 > \omega_2$
(C) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(D) $\omega_1 < \omega_2$

30. A.
Both amplitude and energy get maximised when the frequency is equal to the natural frequency. This is the condition of resonance.
$\omega_1 = \omega_2$

31. One mole of ideal monoatomic gas (γ = 5/30) is mixed with one mole of diatomic gas (γ = 7/5). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.
(A) 3/2
(B) 23/15
(C) 35/23
(D) 4/3

31. A.
$Q = Q_1 + Q_2$

$$\frac{n_1 + n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\gamma_m = \frac{3}{2}$$

32. If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously will be
(A) 4
(B) 16
(C) 32
(D) 64.

32. D.
According to Stefan's law,
$P \propto AT^4$ and $A \propto r^2$
$P \propto r^2T^4$

33. Which of the following statements is correct for any thermodynamic system?
(A) The internal energy changes in all processes.
(B) Internal energy and entropy are state functions.
(C) The change in entropy can never be zero.
(D) The work done in an adiabatic process is always zero.

33. B.

34. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining two vessels is opened, the temperature inside the vessel at equilibrium will be
(A) $T_1 + T_2$
(B) $(T_1 + T_2)/2$
(C) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$
(D) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2T_2T_2}$

34. C.
The number of moles of system remains same
According to Boyle's law,
$P_1V_1 + P_2V_2 = P(V_1 + V_2)$

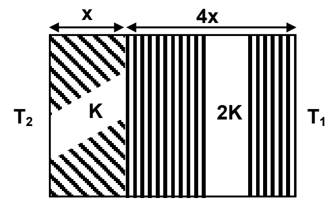
$$\therefore \quad T = \frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

35. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(A) E/c (B) 2E/c
(C) Ec (D) E/c^2

35. B.

$$\Delta P_{surface} = -\Delta P = \frac{2E}{c}.$$

36. The temperature of two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equal to



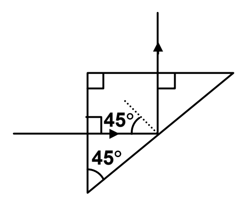
(A) 1 (B) ½
(C) 2/3 (D) 1/3

36. D.

$$\Delta q = \frac{kA}{x}\left[T_2 - \frac{2T_2 - T_1}{3}\right]$$

$$= \frac{kA}{3x}\left[T_2 - T_1\right]$$

37. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n



(A) $n < \frac{1}{2}$ (B) $n > \sqrt{2}$
(C) $n > \frac{1}{\sqrt{2}}$ (D) $n < \sqrt{2}$

37. B.
Angle of incidence i > C for total internal reflection.
Here i = 45° inside the medium.
$\therefore$ $45° > \sin^{-1}(1/n)$
$\Rightarrow$ $n > \sqrt{2}$.

38. A plane convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?
(A) 20 cm (B) 30 cm
(C) 60 cm (D) 80 cm

38. A.

$$\frac{1}{F} = \frac{2}{f_1} + \frac{1}{f_m}$$

and $\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right) = \frac{1}{60}$

and $f_m = 15$ cm.
$\therefore$ F = 10 cm.
Object should be placed at 20 cm from the lens.

39. The angle of incidence at which reflected light totally polarized for reflection from air to glass (refractive index n), is
(A) $\sin^{-1}(n)$ (B) $\sin^{-1}(1/n)$
(C) $\tan^{-1}(1/n)$ (D) $\tan^{-1}(n)$

39. D.
Brewster's law: According to this law the ordinary light is completely polarised in the plane of incidence when it gets reflected from transparent medium at a particular angle known as the angle of polarisation.
$n = \tan i_p$.

40. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
(A) infinite (B) five
(C) three (D) zero

40. B.
For interference maxima, $d \sin \theta = n\lambda$
Here $d = 2\lambda$
$\therefore \sin \theta = n/2$ and is satisfied by 5 integral values of n (–2, –1, 0, 1, 2), as the maximum value of $\sin \theta$ can only be 1.

41. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then
(A) wavelength is doubled and the frequency remains unchanged
(B) wavelength is doubled and frequency becomes half
(C) wavelength is halved and frequency remains unchanged
(D) wavelength and frequency both remain unchanged.

41. C.
Refractive index, $\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = 2$
Speed and wavelength of wave will becomes half, the frequency remaining unchanged (frequency of a wave depends on the source as due to refraction, it is assumed that the energy is conserved. $h\nu$ remains the same)

42. Two spherical conductor B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion, between B and C is
(A) F/4 (B) 3F/4
(C) F/8 (D) 3F/8.

42. D.
$$F' = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(3q/4)}{d^2} = \frac{3F}{8}.$$

43. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v it approaches Q upto a closest distance r and then returns. If q were given a speed 2v, the closest distances of approach would be
(A) r (B) 2r
(C) r/2 (D) r/4

43. D.

By principle of conservation of energy

$$\frac{1}{2}mv^2 = \frac{KqQ}{r} \qquad \ldots(i)$$

Finally, $\frac{1}{2}m(2v)^2 = \frac{KqQ}{r^2} \qquad \ldots(ii)$

Equation (i) ÷ (ii),

$$\frac{1}{4} = \frac{r'}{r}$$

$$\Rightarrow \quad r' = \frac{r}{4}.$$

44. Four charges equal to –Q are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is

(A) $-\frac{Q}{4}(1+2\sqrt{2})$
(B) $\frac{Q}{4}(1+2\sqrt{2})$
(C) $-\frac{Q}{2}(1+2\sqrt{2})$
(D) $\frac{Q}{2}(1+2\sqrt{2})$

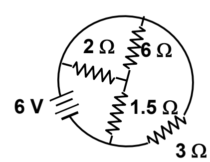
44. B.

q = $+\frac{Q}{4}(1+2\sqrt{2})$

45. Alternating current can not be measured by D.C. ammeter because
(A) A.C. cannot pass through D.C.
(B) A.C. changes direction
(C) average value of current for complete cycle is zero
(D) D.C. ammeter will get damaged.

45. C.

46. The total current supplied to the circuit by the battery is
(A) 1 A
(B) 2 A
(C) 4 A
(D) 6 A



46. C.

The given circuit can be written as

$$I = \frac{6\ V}{1.5\ \Omega} = 4A.$$

47. The resistance of the series combination of two resistances is S. When they are joined in parallel through total resistance is P. If S = nP, then the minimum possible value of n is
(A) 4
(B) 3
(C) 2
(D) 1

47. A.

Let resistances be R_1 and R_2

So, S = $R_1 + R_2$;

$$P = \frac{R_1R_2}{R_1+R_2}$$

S = nP

$$R_1 + R_2 = \frac{nR_1R_2}{R_1+R_2}$$

$$(R_1+R_2)^2 = nR_1R_2$$

If $R_1 = R_2$, so minimum value of n = 4.

48. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the length and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wire will be
(A) 3 (B) 1/3
(C) 8/9 (D) 2.

48. B.

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

[current divider rule since voltage is same in parallel]

$$\frac{I_1}{I_2} = \frac{L_2}{L_1} \times \frac{r_1^2}{r_2^2}$$

$$\therefore \frac{I_1}{I_2} = \frac{3}{4} \times \left(\frac{2}{3}\right)^2 = \frac{1}{3}.$$

49. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If X < Y, then where will be the new position of the null point from the same end, if one decides to balance a resistance of 4X against Y?
(A) 50 cm (B) 80 cm
(C) 40 cm (D) 70 cm

49. A.

We have from meter bridge experiment,

$\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2}$, where $\ell_2 = (100 - \ell_1)$ cm

In the first case, X/Y = 20/80

In the second case $\frac{4X}{Y} = \frac{\ell}{100 - \ell}$

$\ell = 50$ cm.

50. The thermistors are usually made of
(A) metals with low temperature coefficient of resistivity
(B) metals with high temperature coefficient of resistivity
(C) metal oxides with high temperature coefficient of resistivity ‘
(D) semiconducting materials having low temperature coefficient of resistivity.

50. C.

These are devices whose resistance varies quite markedly with temperature mean having high temperature coefficient of resistivity. [Their name are derived from thermal resistors]. Depending on their composition they can have either negative temperature coefficient or positive temperature coefficient or positive temperature coefficient or positive temperature coefficient characteristics.

The negative temperature coefficient types consists of a mixture of oxides of iorn, nickel and cobalt with small amounts of other substance. The positive temperature coefficient types are based on barium titanate.

51. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is
(A) 50 s (B) 100 s
(C) 150 s (D) 200 s

51. C.

Let t be the time taken, then

$\frac{836 \times t}{4.2} = 1000 \times 1 \times (40 - 10)$ [using Q = mst]

$\Rightarrow$ t = 150 sec.

52. The thermo emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\,\theta + b\theta^2$ in volts where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is
(A) 700°C
(B) 350°C
(C) 1400°C
(D) no neutral temperature is possible for this thermocouple.

52. D.

$E = a\theta + b\theta^2$

At neutral temperature $dE/d\theta = 0$

$\therefore\ \frac{dE}{d\theta} = a + 2b\theta_n = 0\,;\ \theta_n = -\frac{a}{2b}$

Now $\frac{a}{b} = 700°C$ (given)

$\theta_n = -700/2 = -350°C$

Now $\theta_c = 0°C$.

So, $\theta_n > 0°C$

But mathematically $\theta_n < 0°C$.

53. The electrochemical equivalent of a metal is 3.3×10^{-7} kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 seconds will be
(A) 19.8×10^{-7} kg
(B) 9.9×10^{-7} kg
(C) 6.6×10^{-7} kg
(D) 1.1×10^{-7} kg

53. A.

m = Zit,

$m = 3.3 \times 10^{-7} \times 3 \times 2 = 19.8 \times 10^{-7}$ kg.

54. A current I ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is
(A) infinite
(B) zero
(C) $\frac{\mu_0}{4\pi}\frac{2i}{r}$ tesla
(D) $\frac{2i}{r}$ tesla

54. B.

Considering Ampere's loop (shown by dotted line), no current is enclosed by this loop. Therefore, the magnetic field will be zero inside the tube.

55. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be
(A) nB
(B) n^2 B
(C) 2nB
(D) $2n^2B$

55. B.

$B' = \frac{n\mu_0 i}{2r'} = n^2\frac{\mu_0 i\pi}{\ell} = n^2B$.

56. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?
(A) 250 μT (B) 150 μT
(C) 125 μT (D) 75 μT

56. A.

Using formula $B = \frac{\mu_0 i R^2}{2(R^2 + X^2)^{3/2}}$, we get

$$54 = \frac{\mu_0 i (3)^2}{2[(3)^2 + (4)^2]^{3/2}} \quad \ldots(i)$$

At the centre of the coil, X = 0 and B = $\frac{\mu_0 i}{2(3)}$

Using equation (i)

$$B = \frac{54 \times 5^3}{(3)^2 \times 3} \Rightarrow B = 250\ \mu T.$$

57. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them increased to two times and its direction reversed. The distance is also increased to 3d. The new value of the force between them is
(A) –2F (B) F/3
(C) –2F/3 (D) –F/3

57. C.

Force between two long conductor carrying current

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell$$

According to question

$$F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{d}\ell$$

From equation (i) and (ii), $F' = -\frac{3}{2}F.$

58. The length of a magnet is large compared to its width and breadth. The time period of its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be
(A) 2 s (B) 2/3 s
(C) 2√3 s (D) 2/√3 s .

58. B.

Time period of vibration, $T = 2\pi\sqrt{\frac{T}{MB}}$

Where ℓ = moment of inertia of magnet, M = magnetic moment

$$I = \frac{m\ell^2}{12} \text{ and } M = \text{pole strength} \times \ell$$

$$I' = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2 \times 3 = \frac{I}{9}$$

and M' = pole strength (will remain the same) × (ℓ/3) × 3 = M.

$$T' = \frac{T}{\sqrt{9}} = \frac{2}{9}s.$$

59. The materials suitable for making electromagnets should have
(A) high retentivity and high coercivity
(B) low retentivity and low coercivity
(C) high retentivity and low coercivity
(D) low retentivity and high coercivity

59. B.

60. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50 V. The voltage across the LC combination will be
(A) 50 V
(B) $50\sqrt{2}$ V
(C) 100 V
(D) 0 V(zero)

60. D.
In series LCR circuit, the voltage across the inductor (L) and the capacitor (C) are in opposite phase.

61. A coil having n turns and resistance 4R Ω. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is
(A) $-\frac{W_2 - W_1}{5Rnt}$
(B) $-\frac{(W_2 - W_1)}{5Rt}$
(C) $-\frac{W_2 - W_1}{Rnt}$
(D) $-\frac{n(W_2 - W_1)}{Rt}$

61. B.

$$I = -\frac{n}{R'}\frac{d\phi}{dt}$$

or, $I = -\frac{1}{R'}n\left[\frac{W_2 - W_1}{t_2 - t_1}\right]$

(W_1 and W_2 are not the magnetic field, but the values of flux associated with one turn of coil)

$$I = \frac{-1}{(R + 4R)}\frac{n(W_2 - W_1)}{t}$$

or, $I = -\frac{n(W_2 - W_1)}{5Rt}$

62. In a uniform magnetic field of induction B a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R the mean power generated per period of rotation is
(A) $\frac{B\pi r^2\omega}{2R}$
(B) $\frac{(B\pi r^2\omega)^2}{2R}$
(C) $\frac{(B\pi r\omega)^2}{2R}$
(D) $\frac{(B\pi r\omega^2)^2}{8R}$

62. B.

Magnetic flux = BA cos θ = $B \cdot \frac{\pi r^2}{2}\cos\omega t$

$$\therefore\ \varepsilon_{ind} = -\frac{d\phi}{dt} = \frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\therefore\ P = \frac{\varepsilon^2_{ind}}{R} = \frac{B^2\pi^2 r^4\omega^2\sin^2\omega t}{4R}$$

Now, $\langle \sin^2\omega t\rangle = \frac{1}{2}$ (mean value)

$$\therefore\ \langle P\rangle = \frac{(B\pi r^2\omega)^2}{8R}.$$

63. In a LCR circuit capacitance is changed from C to 2C. For the resonant frequency to remain unchanged, the inductance should be changed from L to

(A) 4L (B) 2L

(C) L/2 (D) L/4

63. C.

$$\omega_{res} = \frac{1}{\sqrt{LC}}$$

if ω_{res} is to remain same, the product LC should also not change.

$\Rightarrow LC = L'C'$

$\Rightarrow LC = L'2C$

$\Rightarrow L' = L/2$

64. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.3×10^{-4} T, then the e.m.f. developed between the two ends of the conductor is

(A) depends on the nature of the metal used

(B) depends on the intensity of the radiation

(C) depends both on the intensity of the radiation and the metal used

(D) is the same for all metals and independent of the intensity of the radiation.

64. B.

emf. developed is given by

$$\varepsilon_{ind} = \frac{1}{2}B\omega R^2 = 50\ \mu V.$$

65. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal V_s the frequency, of the incident radiation gives straight line whose slope

(A) depends on the nature of the metal used

(B) depends on the intensity of the radiation

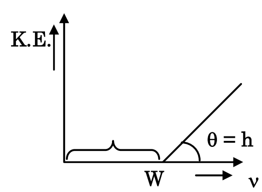
(C) depends both on the intensity of the radiation and the metal used

(D) is the same for all metals and independent of the intensity of the radiation.

65. D.

$KE_{max} = h\nu - W$ {y = mx + C}

Slope of the line in the graph is h, the Planck's constant.



66. The work function of a substance is 4.0 eV. Then longest wavelength of light that can cause photoelectron emission from this substance approximately

(A) 540 nm (B) 400 nm

(C) 310 nm (D) 220 nm

66. C.

$$\frac{hc}{\lambda} = W$$

$$\lambda_{longest} = \frac{hc}{W} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{4.0 \times 1.6 \times 10^{-19}}$$

$\Rightarrow \lambda_{longest} \approx 310$ nm.

67. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge = 9.9×10^{-15} kg and g = 10 m/s^2)
(A) 3.3×10^{-18} C
(B) 3.2×10^{-18} C
(C) 1.6×10^{-18} C
(D) 4.8×10^{-18} C.

67. A.
Since ball is hanging in equilibrium, force by gravity is balanced by electric force.
qE = mg

$$\Rightarrow q = \frac{m \times g}{E}$$

$$\Rightarrow \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4}$$

$\therefore$ q = 3.3×10^{-18} C

68. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be
(A) $2^{1/3} : 1$
(B) $1 : 3^{1/2}$
(C) $3^{1/2} : 1$
(D) $1 : 2^{1/3}$

68. B.

$$\frac{R_1}{R_2} = \left(\frac{m_2}{2m_2}\right)^{1/3}$$

$$\Rightarrow \frac{R_1}{R_2} = 1 : 2^{1/3}.$$

69. The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is
(A) 13.9 MeV
(B) 26.9 MeV
(C) 23.6 MeV
(D) 19.2 MeV

69. C.
Energy released = total binding energy of product – total binding energy of reactants
$\Rightarrow$ 28 – (2 $\times$ 2.2) = 28 – 4.4 = 236 MeV.

70. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
(A) 1 Å
(B) 10^{-10} cm
(C) 10^{-12} cm
(D) 10^{-15} cm

70. C.
At closest approach, all the kinetic energy of the α-particle will converted into the potential energy of the system, K.E. = P.E.

$$5 \text{ MeV} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$5 \times 10^6 \times e = 9 \times 10^9 \frac{Z_1 \times Z_2 e^2}{r}$$

$$r = \frac{9 \times 10^9 \times 92 \times 2 \times 1.6 \times 10^{-19}}{5 \times 10^6}$$

$\therefore$ r = 5.3×10^{-14} m = 5.3×10^{-12} cm.

71. When npn transistor is used as amplifier
(A) electrons move from base to collector
(B) holes move from emitter to base
(C) electrons move from collector to base
(D) holes move from base to emitter.

71. A.
When npn transistor is used, majority charge carrier electrons of n type emitter move from emitter to base and then base to collector.

72. For a transistor amplifier in common emitter configuration having load impedance of 1 kΩ (h_{fe} = 50 and h_{oe} = 25) the current gain is
(A) –5.2
(B) –15.7
(C) –24.8
(D) –48.78

72. D.
In CE configuration, $A_i = \frac{-h_{fe}}{1 + h_{0e}R_L}$

$$= \frac{-50}{1 + 25 \times 10^{-6} \times 1 \times 10^{3}} = -48.78$$

73. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of
(A) each of them increases
(B) each of them decreases
(C) copper decreases and germanium increases
(D) copper increases and germanium decreases.

73. D.
Copper is metallic conductor and germanium is semiconductor therefore as temperature decreases resistance of good conductor decreases while for semiconductor it increases.

74. The manifestation of band structure in solids is due to
(A) Heisenberg's uncertainty principle
(B) Pauli's exclusion principle
(C) Bohr's correspondence principle
(D) Boltzmann's law

74. B.

75. When p-n junction diode is forward biased
(A) the depletion region is reduced and barrier height is increased
(B) the depletion region is widened and barrier height is reduced.
(C) both the depletion region and barrier height reduced
(D) both the depletion region and barrier height increased.

75. C.

AIEEE

76. Which of the following sets of quantum numbers is correct for an electron in 4f orbital?

(1) n = 4, l =3, m = +4, s = $+\frac{1}{2}$
(2) n = 3, l = 2, m = -2, S = $+\frac{1}{2}$
(3) n =4, l = 3, m = +1, s = $+\frac{1}{2}$
(4) n =4, l = 4, m -4, s = $-\frac{1}{2}$

Ans. n =4, l = 3, m = +1, s = $+\frac{1}{2}$

77. Consider the ground state of Cr atom (Z = 24). The number of electrons with the azimuthal quantum numbers l =1 and 2 are respectively

(1) 12 and 4
(2) 16 and 5
(3) 16 and 4
(4) 12 and 5

Ans. 12 and 5

78. Which one the following ions has the highest value of ionic radius?

(1) Li^{+}
(2) F^{-}
(3) O^{2-}
(4) B^{3+}

Ans. O^{2-}

79. The wavelength of the radiation emitted, when in hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant = $1.097 \times 10^{7}\ m^{-1}$)

(1) 91 nm
(2) 9.1×10^{-8} nm
(3) 406 nm
(4) 192 nm

Ans. 91 nm

80. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is

(1) $H_2S < SiH_4 < NH_3 < BF_3$
(2) $H_2S < NH_3 < BF_3 < SiH_4$
(3) $H_2S < NH_3 < SiH_4 < BF_3$
(4) $NH_3 < H_2S < SiH_4 < BF_3$

Ans. $H_2S < NH_3 < SiH_4 < BF_3$

81. Which one the following sets of ions represents the collection of isoelectronic species?

(1) K^{+}, Ca^{2+}, Sc^{3+}, Cl^{-}
(2) Na^{+}, Mg^{2+}, Al^{3+}, Cl^{-}
(3) K^{+}, Cl^{-}, Mg^{2+}, Sc^{3+}
(4) Na^{+}, Ca^{2+}, Sc^{3+}, F^{-}

Ans. K^{+}, Ca^{2+}, Sc^{3+}, Cl^{-}

82. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is

(1) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(2) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$
(3) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(4) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$

Ans. $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$

83. The bond order in NO is 2.5 while that in NO^{+} is 3. Which of the following statements is true for these two species?

(1) Bond length in NO^{+} is greater than in NO
(2) Bond length is unpredictable
(3) Bond length in NO^{+} in equal to that in NO
(4) Bond length in NO is greater than in NO^{+}

Ans. Bond length in NO is greater than in NO^+

84. The formation of the oxide ion $O^{2-}(g)$ requires first an exothermic and then an endothermic step as shown below

$$O(g) + e^- O^-(g) \Delta H^\circ = -142\, kJmol^{-1}$$

$$O^-(g) + e^- O^{2-}(g) \Delta H^\circ = 844\, kJmol^{-1}$$

(1) Oxygen is more electronegative
(2) O^- ion has comparatively larger size than oxygen atom
(3) O^- ion will tend to resist the addition of another electron
(4) Oxygen has high electron affinity

Ans. O^- ion will tend to resist the addition of another electron

85. The states of hybridization of boron and oxygen atoms in boric acid (H_3BO_3) are respectively
(1) sp^2 and sp^2 (2) sp^3 and sp^3
(3) sp^3 and sp^2 (4) sp^2 and sp^3

Ans. sp^2 and sp^3

86. Which one of the following has the regular tetrahedral structure?
(1) XeF_4 (2) $[Ni(CN)_4]^{2-}$
(3) BF_4^- (4) SF_4

Ans. BF_4^-

87. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?
(1) $(n-1)d^8ns^2$ (2) $(n-1)d^5ns^2$
(3) $(n-1)d^3ns^2$ (4) $(n-1)d^5ns^{-1}$

Ans. $(n-1)d^5ns^2$

88. As the temperature is raised from 20°C to 40°C, the average kinetic energy of neon atoms changes by a factor of which of the following?
(1) ½ (2) 2
(3) $\frac{313}{293}$ (4) $\sqrt{\frac{313}{293}}$

Ans. $\frac{313}{293}$

89. The maximum number of 90° angles between bond pair of electrons is observed in
(1) dsp^3 hybridization (2) sp^3d^2 hybridization
(3) dsp^2 hybridization (4) sp^3d hybridization

Ans. sp^3d^2 hybridization

90. Which one of the following aqueous solutions will exhibit highest boiling point?
(1) 0.01 M Na_2SO_4 (2) 0.015 M glucose
(3) 0.015 M urea (4) 0.01 M KNO_3

Ans. 0.01 M Na_2SO_4

91. Which among the following factors is the most important in making fluorine the strongest oxidizing halogen?

(1) Electron affinity
(2) Bond dissociation energy
(3) Hydration enthalpy
(4) Ionization enthalpy

Ans. Bond dissociation energy

92. In Vander Waals equation of state of the gas law, the constant 'b' is a measure of
(1) intermolecular repulsions
(2) intermolecular collisions per unit volume
(3) Volume occupied by the molecules
(4) intermolecular attraction

Ans. Volume occupied by the molecules

93. The conjugate base of $H_2PO_4^-$ is
(1) PO_4^{3-}
(2) HPO_4^{2-}
(3) H_3PO_4
(4) P_2O_5

Ans. HPO_4^{2-}

94. 6.02×10^{20} molecules of urea are present in 100 ml of its solution. The concentration of urea solution is
(1) 0.001 M
(2) 0.1 M
(3) 0.02 M
(4) 0.01 M

Ans. 0.01 M

95. To neutralize completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1 M aqueous KOH solution required is
(1) 10 mL
(2) 60 mL
(3) 40 mL
(4) 20 mL

Ans. 40 mL

96. For which of the following parameters the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values?
(Assume ideal behaviour)
(1) Heat of vaporization
(2) Gaseous densities at the same temperature and pressure
(3) Boiling points
(4) Vapour pressure at the same temperature

Ans. Gaseous densities at the same temperature and pressure

97. Which of the following liquid pairs shows a positive deviation from Raoult's law?
(1) Water – hydrochloric acid
(2) Acetone – chloroform
(3) Water – nitric acid
(4) Benzene – methanol

Ans. Benzene – methanol

98. Which one of the following statements is false?
(1) Raoult's law states that the vapour pressure of a components over a solution is proportional to its mole fraction
(2) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression
(3) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ sucrose
(4) The osmotic pressure (π) = MRT, where M is the molarity of the solution

Ans. Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression

99. What type of crystal defect is indicated in the diagram below?

Na^+ Cl^- $Na^+$$Cl^-$ $Na^+$$Cl^-$
Cl^- □ Cl^- □ Na^+□ Na^+
Na^+ Cl^- □ Cl^- Na^+ Cl^-
Cl^- $Na^+$$Cl^-$ Na^+ □ Na^+

(1) Frenkel defect
(2) Frenkel and Schottky defects
(3) Interstitial defect
(4) Schottky defect

Ans. Schottky defect

100. An ideal gas expands in volume from 1×10^{-3} m^3 to 1×10^{-2} m^3 at 300 K against a constant pressure of 1×10^5 Nm^{-2}. The work done is

(1) -900 J
(2) 900 kJ
(3) 2780 kJ
(4) -900 kJ

Ans. -900 J

101. In hydrogen – oxygen fuel cell, combustion of hydrogen occurs to

(1) generate heat
(2) remove adsorbed oxygen from electrode surfaces
(3) produce high purity water
(4) create potential difference between the two electrodes

Ans. create potential difference between the two electrodes

102. In first order reaction, the concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is

(1) 30 minutes
(2) 60 minutes
(3) 7.5 minutes
(4) 15 minutes

Ans. 30 minutes

103. What is the equilibrium expression for the reaction $P_{4(s)} + 5O_{2(g)} \rightleftharpoons P_4O_{10(s)}$?

(1) $Kc = [P_4O_{10}] / P_4] [O_2]^5$
(2) $Kc = 1/[O_2]^5$
(3) $Kc = [O_2]^5$
(4) $Kc = [P_4O_{10}] / 5[P_4][O_2]$

Ans. $Kc = 1/[O_2]^5$

104. For the reaction, $CO(g) + Cl_2(g) \rightleftharpoons COCl_2(g)$ the $\frac{K_p}{K_c}$ is equal to

(1) $\frac{1}{RT}$
(2) 1.0
(3) $\sqrt{RT}$
(4) RT

Ans. $\frac{1}{RT}$

105. The equilibrium constant for the reaction $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$ at temperature T is 4×10^{-4}. The value of Kc for the reaction $NO(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{1}{2}O_2(g)$ at the same temperature is

(1) 2.5×10^2 (2) 0.02
(3) 4×10^{-4} (4) 50

Ans. 50

106. The rate equation for the reaction $2A + B \longrightarrow C$ is found to be: rate k[A][B]. The correct statement in relation to this reaction is that the
(1) unit of K must be s^{-1}
(2) values of k is independent of the initial concentration of A and B
(3) rate of formation of C is twice the rate of disappearance of A
(4) $t_{1/2}$ is a constant

Ans. values of k is independent of the initial concentration of A and B

107. Consider the following E° values
$E^{\circ}_{Fe^{3+}/Fe^{2+}} = 0.77$ V
$E^{\circ}_{Sn^{2+}/Sn} = -0.14$V
Under standard conditions the potential for the reaction
$Sn(s) + 2Fe^{3+}(aq) \longrightarrow 2Fe^{2+}(aq) + Sn^{2+}(aq)$ is
(1) 1.68 V (2) 0.63 V
(3) 0.91 V (4) 1.40 V

Ans. 0.91 V

108. The molar solubility product is K_{sp}. 's' is given in terms of K_{sp} by the relation
(1) $s = \left(\frac{K_{sp}}{128}\right)^{1/4}$ (2) $s = \left(\frac{K_{sp}}{256}\right)^{1/5}$
(3) $s = (256K_{sp})^{1/5}$ (4) $s = (128K_{sp})^{1/4}$

Ans. $s = \left(\frac{K_{sp}}{256}\right)^{1/5}$

109. The standard e.m.f of a cell, involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol^{-1}: R = 8.314 JK^{-1} mol^{-1})
(1) 1.0×10^1 (2) 1.0×10^{30}
(3) 1.0×10^{10} (4) 1.0×10^5

Ans. 1.0×10^{10}

110. The enthalpies of combustion of carbon and carbon monoxide are -393.5 and -283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is
(1) 110.5 kJ (2) -110.5 kJ
(3) -676.5 kJ (4) 676.5 kJ

Ans. -110.5 kJ

111. The limiting molar conductivities $\Lambda°$ for NaCl, KBr and KCl are 126, 152 and 150 S cm^2 mol^{-1} respectively. The $\Lambda°$ for NaBr is
(1) 128 S cm^2 mol^{-1} (2) 302 S cm^2 mol^{-1}
(3) 278 S cm^2 mol^{-1} (4) 176 S cm^2 mol^{-1}

Ans. 128 S cm^2 mol^{-1}

112. In a cell that utilises the reaction $Zn(s) + 2H^+(aq) \longrightarrow Zn^{2+}(aq) + H_2(g)$ addition of H_2SO_4 to cathode compartment, will
(1) lower the E and shift equilibrium to the left
(2) increases the E and shift equilibrium to the left
(3) increase the E and shift equilibrium to the right
(4) Lower the E and shift equilibrium to the right

Ans. increase the E and shift equilibrium to the right

113. Which one the following statement regarding helium is incorrect?
(1) It is used to fill gas balloons instead of hydrogen because it is lighter and non – inflammable
(2) It is used in gas – cooled nuclear reactors
(3) It is used to produce and sustain powerful superconducting reagents
(4) It is used as cryogenic agent for carrying out experiments at low temperatures

Ans. It is used to fill gas balloons instead of hydrogen because it is lighter and non – inflammable

114. Identify the correct statements regarding enzymes
(1) Enzymes are specific biological catalysts that can normally function at very high temperature (T ~ 1000 K)
(2) Enzymes are specific biological catalysts that the posses well – defined active sites
(3) Enzymes are specific biological catalysts that can not be poisoned
(4) Enzymes are normally heterogeneous catalysts that are very specific in their action

Ans. Enzymes are specific biological catalysts that the posses well – defined active sites

115. One mole of magnesium nitride on the reaction with an excess of water gives
(1) one mole of ammonia
(2) two moles of nitric acid
(3) two moles of ammonia
(4) one mole of nitric acid

Ans. two moles of ammonia

116. Which one of the following ores is best concentrated by froth – floatation method?
(1) Magnetite
(2) Malachite
(3) Galena
(4) Cassiterite

Ans. Galena

117. Beryllium and aluminium exhibit many properties which are similar. But the two elements differ in
(1) exhibiting maximum covalency in compound
(2) exhibiting amphoteric nature in their oxides
(3) forming covalent halides
(4) forming polymeric hydrides

Ans. exhibiting maximum covalency in compound

118. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives
(1) $Al^{3+} + 3Cl^-$
(2) $Al_2O_3 + 6HCl$
(3) $[Al(OH)_6]^{3-}$
(D) $[Al(H_2O)_6]^{3+} + 3Cl^-$

Ans. $[Al(H_2O)_6]^{3+} + 3Cl^-$

119. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. White metallic tin buttons got converted to grey powder. This transformation is related to
(1) an interaction with nitrogen of the air at very low temperatures
(2) an interaction with water vapour contained in the humid air
(3) a change in the partial pressure of oxygen in the air
(4) a change in the crystalline structure of tin

Ans. a change in the crystalline structure of tin

120. The $E^{\circ}_{M^{+3}/M^{2+}}$ values for Cr, Mn, Fe and Co are – 0.41, +1.57, + 0.77 and +1.97 V respectively. For which one of these metals the change in oxidation state form +2 to +3 is easiest?
(1) Cr (2) Co
(3) Fe (4) Mn

Ans. Cr

121. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction?
(1) Cu_2I_2 is reduced (2) Evolved I_2 is reduced
(3) $Na_2S_2O_3$ is oxidized (4) CuI_2 is formed

Ans. CuI_2 is formed

122. Among the properties (a) reducing (b) oxidising (c) complexing, the set of properties shown by CN^- ion towards metal species is
(1) a, b (2) a, b, c
(3) c, a (4) b, c

Ans. c, a

123. The coordination number of central metal atom in a complex is determined by
(1) the number of ligands around a metal ion bonded by sigma bonds
(2) the number of only anionic ligands bonded to the metal ion
(3) the number of ligands around a metal ion bonded by sigma and pi- bonds both
(4) the number of ligands around a metal ion bonded by pi-bonds

Ans. the number of ligands around a metal ion bonded by sigma

124. Which one of the following complexes in an outer orbital complex?
(1) $[Fe(CN)_6]^{4-}$ (2) $[Ni(NH_3)_6]^{2+}$
(3) $[Co(NH_3)_6]^{3+}$ (4) $[Mn(CN)_6]^{4-}$

Ans. $[Ni(NH_3)_6]^{2+}$

125. Coordination compound have great importance in biological systems. In this context which of the following statements is incorrect?
(1) Chlorophylls are green pigments in plants and contains calcium
(2) Carboxypeptidase – A is an enzyme and contains zinc
(3) Cyanocobalamin is B_{12} and contains cobalt
(4) Haemoglobin is the red pigment of blood and contains iron

Ans. Chlorophylls are green pigments in plants and contains calcium

126. Cerium (Z = 58) is an important member of the lanthanoids. Which of the following statements about cerium is incorrect?
(1) The common oxidation states of cerium are +3 and +4
(2) Cerium (IV) acts as an oxidizing agent
(3) The +4 oxidation state of cerium is not known in solutions
(4) The +3 oxidation state of cerium is more stable than the +4 oxidation state

Ans. The +4 oxidation state of cerium is not known in solutions

127. Which one the following has largest number of isomers?
(1) $[Ru(NH_3)_4Cl_2^+]$ (2) $[Co(en)_2Cl_2]^+$
(3) $[Ir(PR_3)_2\, H(CO)]^{2+}$ (4) $[Co(NH_3)_5Cl]^{2+}$
(R -= alkyl group, en = ethylenediamine)

Ans. $[Co(en)_2Cl_2]^+$

128. The correct order of magnetic moments (spin only values in B.M.) among is
(1) $[MnCl_4]^{2-} > [CoCl_4]^{-2} > [Fe(CN)_6]^{-4}$ (2) $[Fe(CN)_6]^{-4} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(3) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$ (4) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(Atomic numbers: Mn = 25; Fe = 26, Co =27)

Ans. $[MnCl_4]^{2-} > [CoCl_4]^{-2} > [Fe(CN)_6]^{-4}$

129. Consider the following nuclear reactions
$${}^{238}_{92}M \rightarrow {}^{x}_{y}N + {}^{4}_{2}He$$
$${}^{x}_{y}N \rightarrow {}^{A}_{B}L + 2\beta^{+}$$
The number of neutrons in the element L is
(1) 142 (2) 146
(3) 140 (4) 144

Ans. 144

130. The half – life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is
(1) 1.042 g (2) 4.167 g
(3) 3.125 g (4) 2.084 g

Ans. 3.125 g

131. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is
(1) $Fe_4[Fe(CN)_6]_3$ (2) $Na_4[Fe(CN)_5NOS]$
(3) $Fe(CN)_3$ (4) $Na_3[Fe(CN)_6]$

Ans. $Fe_4[Fe(CN)_6]_3$

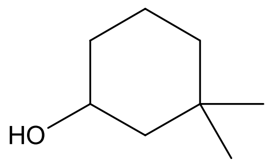
132. The ammonia evolved from the treatment of 0.30 g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution hydroxide solutio for complete neutralization. The organic compound is
(1) acetamide (2) thiourea
(3) urea (4) benzamide

Ans. urea

133. Which one of the following has the minimum boiling point?
(1) n-butane (2) isobutane
(3) 1- butene (4) 1- butyne

Ans. isobutane

134. The IUPAC name of the compound



(1) 3, 3- dimethyl -1- hydroxy cyclohexane (2) 1,1 – dimethyl -3- cyclohexanol
(3) 3,3- dimethyl -1- cyclohexanol (4) 1,1 – dimethyl -3- hydroxy cyclohexane

Ans. 3,3- dimethyl -1- cyclohexanol

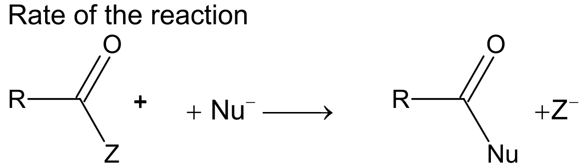
135. Which one the following does not have sp^2 hybridized carbon?
(1) Acetone (2) Acetamide
(3) Acetonitrile (4) Acetic acid

Ans. Acetonitrile

136. Which of the following will have meso-isomer also?
(1) 2- chlorobutane (2) 2- hydroxyopanoic acid
(3) 2,3 – dichloropentane (4) 2-3- dichlorobutane

Ans. 2-3- dichlorobutane

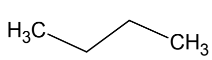
137. Rate of the reaction

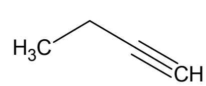


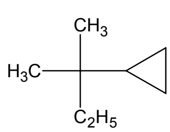
is fastest when Z is
(1) Cl (2) $OCOCH_3$
(3) OC_2H_5 (4) NH_2

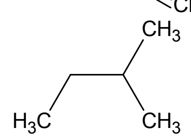
Ans. Cl

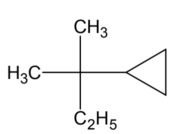
138. Amongst the following compound, the optically active alkane having lowest molecular mass is

(1) 

(2) 

(3) 

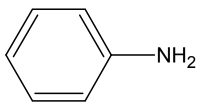
(4) 

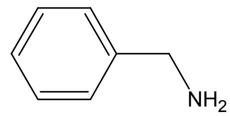
Ans. 

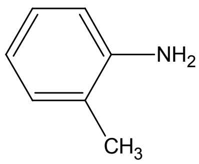
139. Consider the acidity of the carboxylic acids:
(1) PhCOOH
(2) o – $NO_2C_6H_4COOH$
(3) p – $NO_2C_6H_4COOH$
(4) m – $NO_2C_6H_4COOH$

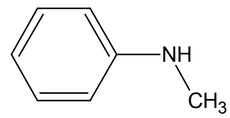
Ans. o – $NO_2C_6H_4COOH$

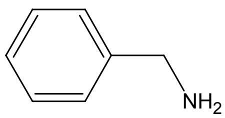
140. Which of the following is the strongest base?

(1) 

(2) 

(3) 

(4) 

Ans. 

141. Which base is present in RNA but not in DNA?
(1) Uracil
(2) Thymine
(3) Guanine
(4) Cytosine

Ans. Uracil

142. The compound formed on heating chlorobenzene with chloral in the presence concentrated sulphuric acid is
(1) gammexene
(2) hexachloroethane
(3) Freon
(4) DDT

Ans. DDT

143. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is
(1) $CH_3COOC_2H_5$ + NaCl
(2) CH_3Cl + C_2H_5COONa
(3) CH_3COCl + C_2H_5OH + NaOH
(4) CH_3COONa + C_2H_5OH

Ans. $CH_3COOC_2H_5$ + NaCl

144. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl given
(1) acetone
(2) acetyl iodide
(3) 2- methyl -2- propanol
(4) acetamide

Ans. 2- methyl -2- propanol

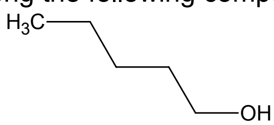
145. Which one of the following reduced with zinc and hydrochloric acid to give the corresponding hydrocarbon?
(1) Ethyl acetate
(2) Butan -2-one
(3) Acetamide
(4) Acetic acid

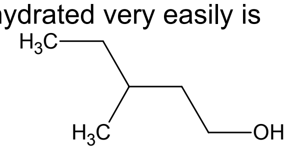
Ans. Butan -2-one

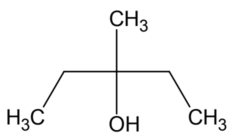
146. Which of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid?
(1) Phenol
(2) Benzoic acid
(3) Butanal
(4) Benzaldehyde

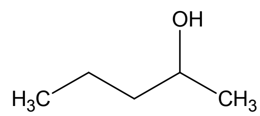
Ans. Benzaldehyde

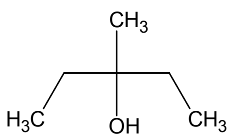
147. Among the following compound which can be dehydrated very easily is

(1)


(2)


(3)


(D)


Ans.


148. Which of the following compound is not chiral?
(1) 1- chloropentane
(2) 3-chloro-2- methyl pentane
(3) 1-chloro -2- methyl pentane
(4) 2- chloropentane

Ans. 1- chloropentane

149. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories?
(1) A co- enzyme
(2) An antibiotic
(3) An enzyme
(4) A hormone

Ans. A hormone

150. The smog is essentially caused by the presence of
(1) O_2 and O_3
(2) O_3 and N_2
(3) Oxides of sulphur and nitrogen
(4) O_2 and N_2

Ans. Oxides of sulphur and nitrogen

SOLUTIONS (AIEEE)

76.	**(3)**	77.	**(4)**	78.	**(3)**	79.	**(1)**
80.	**(3)**	81.	**(1)**	82.	**(2)**	83.	**(4)**
84.	**(3)**	85.	**(4)**	86.	**(3)**	87.	**(2)**
88.	**(3)**	89.	**(2)**	90.	**(1)**	91.	**(2)**
92.	**(3)**	93.	**(2)**	94.	**(4)**	95.	**(3)**
96.	**(2)**	97.	**(4)**	98.	**(2)**	99.	**(4)**
100.	**(1)**	101.	**(4)**	102.	**(1)**	103.	**(2)**
104.	**(1)**	105.	**(4)**	106.	**(2)**	107.	**(3)**
108.	**(2)**	109.	**(3)**	110.	**(2)**	111.	**(1)**
112.	**(3)**	113.	**(1)**	114.	**(2)**	115.	**(3)**
116.	**(3)**	117.	**(1)**	118.	**(4)**	119.	**(4)**
120.	**(1)**	121.	**(4)**	122.	**(3)**	123.	**(1)**
124.	**(2)**	125.	**(1)**	126.	**(3)**	127.	**(2)**
128.	**(1)**	129.	**(4)**	130.	**(3)**	131.	**(1)**
132.	**(3)**	133.	**(2)**	134.	**(2)**	135.	**(3)**
136.	**(4)**	137.	**(1)**	138.	**(3)**	139.	**(2)**
140.	**(2)**	141.	**(1)**	142.	**(4)**	143.	**(1)**
144.	**(3)**	145.	**(2)**	146.	**(4)**	147.	**(3)**
148.	**(1)**	149.	**(4)**	150.	**(3)**		

SOLUTION

76. $4f \longrightarrow n = 4$
$l = 3$
$m = -l \text{ to } +l$
$-3 \text{ to } +3$

77. $24 \longrightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^5$
$l = 1 \rightarrow p \longrightarrow 12$
$l = 2 \rightarrow d \longrightarrow 5$

78. Li^+ F^- O^{-2} B^{+3}

e	2	10	10	2
p	3	9	8	5

79. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= 1.097 \times 10^7 \left(\frac{1}{1}\right)$

$\lambda = \frac{1}{1.097} \times 10^{-7} m$

80. $H_2S \longrightarrow sp^3$
$NH_3 \longrightarrow sp^3$
$BF_3 \longrightarrow sp^2$
$SiH_4 \longrightarrow sp^3$

82. Al, Si, P, S acidity of oxides increases

83. Bond order of NO = 2.5
Bond order of NO^+ = 3
Higher the bond order shorter is the bond length

84. $O^{-1}(g) + e \longrightarrow O^{-2}(g)$
Due to the electronic repulsion, amount of the energy is needed to add electron

86. Total no of valence electrons
$= 3+7\times4+1 = 32$
Total No of hybrid orbital = 4
$\because$ Hybridisation = sp^3

88. $\frac{E_1}{E_2} = \frac{T_1}{T_2}$
$\frac{E_1}{E_2} = \frac{293}{313}$
$\because$ factor = $\frac{313}{293}$

89. sp^3d^2 hybridisation confirms to octahedral or square bipyramidal configuration
$\therefore$ all the bond angles are 90° in the structure

90. Von't Hoffs factor (i) for Na_2SO_4 is maximum i.e. 3(maximum no of particles)
$Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{--}$

92. In Vander Waals equation 'b' is the excluded volume i.e. the volume occupied by the molecules

93. $\because$ $6.02\times10^{+20}$ molecules of urea is present in = $\frac{0.0001\times1000}{100} = 0.01M$

95. No. of gm equivalents of phosphorous acid
= No. of gm equivalents of KOH
$20\times0.1\times2$ (n = factor) $= 0.1 \times V$
$= 0.1 \times V$

$V = \frac{4}{0.1} = 40\,ml$

96. $\because$ the molecular weight of C_2H_5OH & CH_3OCH_3 are same so in its vapour phase at same temperature & pressure the densities will be same

97. Benzene in methanol breaks the H – bonding of the alcohol making its boiling point decrease & there by its vapour pressure increases leading two +ve deviation.

100. Work done = $-P(\Delta V)$
$= -1\times10^5\,[10^{-2} - 10^{-3}] = -900$ J

102. $t_{1/2}$ = 15 minutes
$\therefore$ No. of half lives s =2
($\therefore$ for change of 0.1 to 0.025)
is 30 minutes

103. Applying law of mass action

104. $Kp = Kc\,(RT)^{\Delta n}$

105. As per property of equilibria reverse the equation & divide it by 2

107. $E_{cell} = E^\circ_{RHS} - E^\circ_{LHS}$
= (0.77) – (-0.14)
= 0.91 V

108. $Ksp = 108s^5$
$1\times4^4\times s^{1+4} = 256\,s^5 = Ksp$

109. $\therefore \log K_{eq} = \frac{nE^\circ}{0.0591} = \frac{1\times0.591}{0.0591}$
$\Rightarrow K_{eq} = 10^{10}$

110. $C + O_2 \longrightarrow CO_2$ $\Delta H = -393.5$ kJ
$2CO + \frac{1}{2}O_2 \longrightarrow 2CO_2$ $\Delta H = -283$ kJ
$2C + O_2 \longrightarrow 2CO$ $\Delta H = -110$ kJ

111. $\Lambda^\circ_{NaCl} = \lambda^\circ_{Na} + \lambda^\circ_{Cl} = 126$ … (1)
$\Lambda^\circ_{KBr} = \lambda^\circ_{K^+} + \lambda^\circ_{Br^-} = 152$ … (2)
$\Lambda^\circ_{KCl} = \lambda^\circ_{K^+} + \lambda^\circ_{Cl^-} = 150$ … (3)
$\Lambda^\circ_{NaBr} = \lambda^\circ_{Na} + \lambda^\circ_{Br^-}$
$\Lambda^\circ_{NaBr} = 126 + 152 - 150 = 128$

115. $Mg_3N_2 + 6H_2O \longrightarrow 3Mg(OH)_2 + 2NH_3$

117. $\because$ Be & Al have diagonal relationship & so possess similar properties but Be cannot form polymeric hydrides

120. $\because$ oxidation of potential of Cr is least & so it changes easily from +2 to +3 state

121. $2\,CuSO_4 + 4KI\ (\text{excess}) \longrightarrow 2K_2SO_4 + Cu_2\,I_2 + I_2\uparrow$

$Na_2S_2O_3 + I_2 \longrightarrow Na_2S_4O_6 + 2NaI$

124. sp^3d^2 $\therefore$ outer orbital octahedral complex

125. Chlorophyll contains magnesium instead of calcium

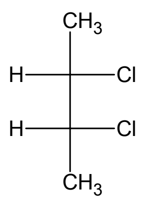
126. Oxidation potential of Ce(IV) in aqueous solution is supposed to be –ve i.e. -0.784 V at 25°C

130. $2^6 = \dfrac{200}{a - x}$

$(a - x) = 3.125$ gm

135. It is having only sp^3 & sp hybridized carbon atom

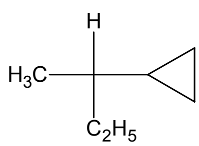
136.



plane symmetry

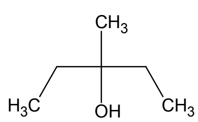
137. Rate of reaction will be fastest when Z is Cl because it is a weakest base

138.



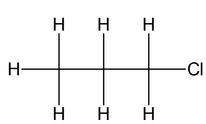
146. Benzaldehyde does not contain α - hydrogen. Hence goes for cannizarro's reaction forming alcohol and acid

147.



Tertiory alcohols will undergo more easily dehydration than secondary & primary

148.



No. chiral centre
Hence not chiral compound

149. Insulin

AIEEE – 2004 (MATHEMATICS)

Important Instructions:

i) *The test is of $1\frac{1}{2}$ hours duration.*

ii) *The test consists of 75 questions.*

iii) *The maximum marks are 225.*

iv) *For each correct answer you will get 3 marks and for a wrong answer you will get -1 mark.*

---

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is
   (1) a function (2) reflexive
   (3) not symmetric (4) transitive

2. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
   (1) {1, 2, 3} (2) {1, 2, 3, 4, 5}
   (3) {1, 2, 3, 4} (4) {1, 2, 3, 4, 5, 6}

3. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals
   (1) $\frac{\pi}{4}$ (2) $\frac{5\pi}{4}$
   (3) $\frac{3\pi}{4}$ (4) $\frac{\pi}{2}$

4. If z = x – i y and $z^{\frac{1}{3}} = p + iq$, then $\frac{\left(\frac{x}{p} + \frac{y}{q}\right)}{\left(p^2 + q^2\right)}$ is equal to
   (1) 1 (2) -2
   (3) 2 (4) -1

5. If $\left|z^2 - 1\right| = |z|^2 + 1$, then z lies on
   (1) the real axis (2) an ellipse
   (3) a circle (4) the imaginary axis.

6. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
   (1) A is a zero matrix (2) $A^2 = I$
   (3) A^{-1} does not exist (4) $A = (-1)I$, where I is a unit matrix

7. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} (10) B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

(1) -2 (2) 5
(3) 2 (4) -1

8. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the value of the determinant

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is

(1) 0 (2) -2
(3) 2 (4) 1

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

(1) $x^2 + 18x + 16 = 0$ (2) $x^2 - 18x - 16 = 0$
(3) $x^2 + 18x - 16 = 0$ (4) $x^2 - 18x + 16 = 0$

10. If (1 – p) is a root of quadratic equation $x^2 + px + (1-p) = 0$, then its roots are

(1) 0, 1 (2) -1, 2
(3) 0, -1 (4) -1, 1

11. Let $S(K) = 1 + 3 + 5 + ... + (2K-1) = 3 + K^2$. Then which of the following is true?

(1) S(1) is correct
(2) Principle of mathematical induction can be used to prove the formula
(3) $S(K) \not\Rightarrow S(K+1)$
(4) $S(K) \Rightarrow S(K+1)$

12. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

(1) 120 (2) 480
(3) 360 (4) 240

13. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(1) 5 (2) ${}^8C_3$
(3) 3^8 (4) 21

14. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is

(1) $\frac{49}{4}$ (2) 4
(3) 3 (4) 12

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals

(1) $-\frac{5}{3}$
(2) $\frac{3}{5}$
(3) $\frac{-3}{10}$
(4) $\frac{10}{3}$

16. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(1) (n – 1)
(2) $(-1)^n(1-n)$
(3) $(-1)^{n-1}(n-1)^2$
(4) $(-1)^{n-1}n$

17. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to

(1) $\frac{1}{2}n$
(2) $\frac{1}{2}n-1$
(3) n – 1
(4) $\frac{2n-1}{2}$

18. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals

(1) 0
(2) 1
(3) $\frac{1}{mn}$
(4) $\frac{1}{m}+\frac{1}{n}$

19. The sum of the first n terms of the series $1^2+2\cdot 2^2+3^2+2\cdot 4^2+5^2+2\cdot 6^2+\ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(1) $\frac{3n(n+1)}{2}$
(2) $\frac{n^2(n+1)}{2}$
(3) $\frac{n(n+1)^2}{4}$
(4) $\left[\frac{n(n+1)}{2}\right]^2$

20. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots$ is

(1) $\frac{(e^2-1)}{2}$
(2) $\frac{(e-1)^2}{2e}$
(3) $\frac{(e^2-1)}{2e}$
(4) $\frac{(e^2-2)}{e}$

21. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is

(1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$

(3) $\frac{6}{65}$ (4) $-\frac{6}{65}$

22. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by

(1) $2(a^2+b^2)$ (2) $2\sqrt{a^2+b^2}$

(3) $(a+b)^2$ (4) $(a-b)^2$

23. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

(1) 60° (2) 90°

(3) 120° (4) 150°

24. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meter away from the tree the angle of elevation becomes 30°. The breadth of the river is

(1) 20 m (2) 30 m

(3) 40 m (4) 60 m

25. If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(1) [0, 3] (2) [-1, 1]

(3) [0, 1] (4) [-1, 3]

26. The graph of the function y = f(x) is symmetrical about the line x = 2, then

(1) f(x + 2)= f(x – 2) (2) f(2 + x) = f(2 – x)

(3) f(x) = f(-x) (4) f(x) = - f(-x)

27. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(1) [2, 3] (2) [2, 3)

(3) [1, 2] (4) [1, 2)

28. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(1) $a \in \underline{\underline{R}}, b \in \underline{\underline{R}}$ (2) $a = 1, b \in \underline{\underline{R}}$

(3) $a \in \underline{\underline{R}}, b = 2$ (4) a = 1 and b = 2

29. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(1) 1
(2) $\frac{1}{2}$
(3) $-\frac{1}{2}$
(4) -1

30. If $x = e^{y+e^{y+\ldots to\,\infty}}$, x > 0, then $\frac{dy}{dx}$ is

(1) $\frac{x}{1+x}$
(2) $\frac{1}{x}$
(3) $\frac{1-x}{x}$
(4) $\frac{1+x}{x}$

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(1) (2, 4)
(2) (2, -4)
(3) $\left(\frac{-9}{8}, \frac{9}{2}\right)$
(4) $\left(\frac{9}{8}, \frac{9}{2}\right)$

32. A function y = f(x) has a second order derivative f″(x) = 6(x – 1). If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x – 5, then the function is

(1) $(x-1)^2$
(2) $(x-1)^3$
(3) $(x+1)^3$
(4) $(x+1)^2$

33. The normal to the curve x = a(1 + cosθ), y = asinθ at 'θ' always passes through the fixed point

(1) (a, 0)
(2) (0, a)
(3) (0, 0)
(4) (a, a)

34. If 2a + 3b + 6c =0, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(1) (0, 1)
(2) (1, 2)
(3) (2, 3)
(4) (1, 3)

35. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(1) e
(2) e – 1
(3) 1 – e
(4) e + 1

36. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is

(1) (sinα, cosα)
(2) (cosα, sinα)
(3) (- sinα, cosα)
(4) (- cosα, sinα)

37. $\int \frac{dx}{\cos x - \sin x}$ is equal to

(1) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$ (2) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(3) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$ (4) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

38. The value of $\int_{-2}^{3}|1-x^2|dx$ is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{7}{3}$ (4) $\frac{1}{3}$

39. The value of I = $\int_{0}^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(1) 0 (2) 1

(3) 2 (4) 3

40. If $\int_{0}^{\pi} x f(\sin x)dx = A\int_{0}^{\pi/2} f(\sin x)dx$, then A is

(1) 0 (2) π

(3) $\frac{\pi}{4}$ (4) 2π

41. If f(x) = $\frac{e^x}{1+e^x}$, I_1 = $\int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and I_2 = $\int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$ then the value of $\frac{I_2}{I_1}$ is

(1) 2 (2) –3

(3) –1 (4) 1

42. The area of the region bounded by the curves y = |x – 2|, x = 1, x = 3 and the x-axis is

(1) 1 (2) 2

(3) 3 (4) 4

43. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is

(1) $2(x^2-y^2)y'=xy$ (2) $2(x^2+y^2)y'=xy$

(3) $(x^2-y^2)y'=2xy$ (4) $(x^2+y^2)y'=2xy$

44. The solution of the differential equation y dx + (x + x^2y) dy = 0 is

(1) $-\frac{1}{xy}=C$ (2) $-\frac{1}{xy}+\log y=C$

(3) $\frac{1}{xy}+\log y=C$ (4) log y = Cx

45. Let A (2, –3) and B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line
(1) 2x + 3y = 9
(2) 2x – 3y = 7
(3) 3x + 2y = 5
(4) 3x – 2y = 3

46. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is
(1) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(2) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(3) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$
(4) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

47. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product, then c has the value
(1) 1
(2) –1
(3) 2
(4) –2

48. If one of the lines given by $6x^2-xy+4cy^2=0$ is 3x + 4y = 0, then c equals
(1) 1
(2) –1
(3) 3
(4) –3

49. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is
(1) $2ax+2by+(a^2+b^2+4)=0$
(2) $2ax+2by-(a^2+b^2+4)=0$
(3) $2ax-2by+(a^2+b^2+4)=0$
(4) $2ax-2by-(a^2+b^2+4)=0$

50. A variable circle passes through the fixed point A (p, q) and touches x-axis. The locus of the other end of the diameter through A is
(1) $(x-p)^2=4qy$
(2) $(x-q)^2=4py$
(3) $(y-p)^2=4qx$
(4) $(y-q)^2=4px$

51. If the lines 2x + 3y + 1 = 0 and 3x – y – 4 = 0 lie along diameters of a circle of circumference 10π, then the equation of the circle is
(1) $x^2+y^2-2x+2y-23=0$
(2) $x^2+y^2-2x-2y-23=0$
(3) $x^2+y^2+2x+2y-23=0$
(4) $x^2+y^2+2x-2y-23=0$

52. The intercept on the line y = x by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is
(1) $x^2+y^2-x-y=0$
(2) $x^2+y^2-x+y=0$
(3) $x^2+y^2+x+y=0$
(4) $x^2+y^2+x-y=0$

53. If $a \neq 0$ and the line 2bx + 3cy + 4d = 0 passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then
(1) $d^2+(2b+3c)^2=0$
(2) $d^2+(3b+2c)^2=0$
(3) $d^2+(2b-3c)^2=0$
(4) $d^2+(3b-2c)^2=0$

54. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is

(1) $3x^2 + 4y^2 = 1$
(2) $3x^2 + 4y^2 = 12$
(3) $4x^2 + 3y^2 = 12$
(4) $4x^2 + 3y^2 = 1$

55. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(1) $\frac{2}{3}$
(2) $\frac{1}{5}$
(3) $\frac{3}{5}$
(4) $\frac{2}{5}$

56. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is

(1) $\frac{3}{2}$
(2) $\frac{5}{2}$
(3) $\frac{7}{2}$
(4) $\frac{9}{2}$

57. A line with direction cosines proportional to 2, 1, 2 meets each of the lines x = y + a = z and x + a = 2y = 2z. The co-ordinates of each of the point of intersection are given by

(1) (3a, 3a, 3a), (a, a, a)
(2) (3a, 2a, 3a), (a, a, a)
(3) (3a, 2a, 3a), (a, a, 2a)
(4) (2a, 3a, 3a), (2a, a, a)

58. If the straight lines x = 1 + s, y = –3 – λs, z = 1 + λs and x = $\frac{t}{2}$, y = 1 + t, z = 2 – t with parameters s and t respectively, are co-planar then λ equals

(1) –2
(2) –1
(3) $-\frac{1}{2}$
(4) 0

59. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

(1) x – y – z = 1
(2) x – 2y – z = 1
(3) x – y – 2z = 1
(4) 2x – y – z = 1

60. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals

(1) $\lambda\vec{a}$
(2) $\lambda\vec{b}$
(3) $\lambda\vec{c}$
(4) 0

61. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The work done in standard units by the forces is given by

(1) 40 (2) 30
(3) 25 (4) 15

62. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\bar{a}+2\bar{b}+3\bar{c},\ \lambda\bar{b}+4\bar{c}$ and $(2\lambda-1)\bar{c}$ are non-coplanar for
(1) all values of λ (2) all except one value of λ
(3) all except two values of λ (4) no value of λ

63. Let $\bar{u}, \bar{v}, \bar{w}$ be such that $|\bar{u}|=1,\ |\bar{v}|=2,\ |\bar{w}|=3$. If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other then $|\bar{u}-\bar{v}+\bar{w}|$ equals
(1) 2 (2) $\sqrt{7}$
(3) $\sqrt{14}$ (4) 14

64. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a}\times\bar{b})\times\bar{c}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then sin θ equals
(1) $\frac{1}{3}$ (2) $\frac{\sqrt{2}}{3}$
(3) $\frac{2}{3}$ (4) $\frac{2\sqrt{2}}{3}$

65. Consider the following statements:
(a) Mode can be computed from histogram
(b) Median is not independent of change of scale
(c) Variance is independent of change of origin and scale.
Which of these is/are correct?
(1) only (a) (2) only (b)
(3) only (a) and (b) (4) (a), (b) and (c)

66. In a series of 2n observations, half of them equal a and remaining half equal –a. If the standard deviation of the observations is 2, then |a| equals
(1) $\frac{1}{n}$ (2) $\sqrt{2}$
(3) 2 (4) $\frac{\sqrt{2}}{n}$

67. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is
(1) $\frac{3}{20}$ (2) $\frac{1}{5}$
(3) $\frac{7}{20}$ (4) $\frac{4}{5}$

68. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability P ($E \cup F$) is

(1) 0.87 (2) 0.77
(3) 0.35 (4) 0.50

69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(1) $\frac{37}{256}$ (2) $\frac{219}{256}$
(3) $\frac{128}{256}$ (4) $\frac{28}{256}$

70. With two forces acting at a point, the maximum effect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are

(1) $(2+\sqrt{2})$N and $(2-\sqrt{2})$N (2) $(2+\sqrt{3})$N and $(2-\sqrt{3})$N
(3) $\left(2+\frac{1}{2}\sqrt{2}\right)$N and $\left(2-\frac{1}{2}\sqrt{2}\right)$N (4) $\left(2+\frac{1}{2}\sqrt{3}\right)$N and $\left(2-\frac{1}{2}\sqrt{3}\right)$N

71. In a right angle ΔABC, $\angle A = 90^\circ$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is

(1) 3 (2) 4
(3) 5 (4) 9

72. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is

(1) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$ (2) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$
(3) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$ (4) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

73. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively

(1) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h (2) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h
(3) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h (4) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

74. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is

(1) $\frac{1}{8}$ m/s (2) $\frac{1}{4}(\sqrt{3}-1)$ m/s
(3) $\frac{1}{4}$ m/s (4) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s

75. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to

(1) $\frac{u^2}{g}$ (2) $\frac{4u^2}{g^2}$

(3) $\frac{u^2}{2g}$ (4) 1

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ANSWERS

1.	**3**	16.	**2**	31.	**4**	46.	**4**	61.	**1**
2.	**1**	17.	**1**	32.	**2**	47.	**3**	62.	**3**
3.	**3**	18.	**1**	33.	**1**	48.	**4**	63.	**3**
4.	**2**	19.	**2**	34.	**1**	49.	**2**	64.	**4**
5.	**4**	20.	**2**	35.	**2**	50.	**1**	65.	**3**
6.	**2**	21.	**1**	36.	**2**	51.	**1**	66.	**3**
7.	**2**	22.	**4**	37.	**4**	52.	**1**	67.	**3**
8.	**1**	23.	**3**	38.	**1**	53.	**1**	68.	**2**
9.	**4**	24.	**1**	39.	**3**	54.	**2**	69.	**4**
10.	**3**	25.	**4**	40.	**2**	55.	**3**	70.	**3**
11.	**4**	26.	**2**	41.	**1**	56.	**3**	71.	**3**
12.	**3**	27.	**2**	42.	**1**	57.	**2**	72.	**1**
13.	**4**	28.	**2**	43.	**3**	58.	**1**	73.	**1**
14.	**1**	29.	**3**	44.	**2**	59.	**4**	74.	**4**
15.	**3**	30.	**3**	45.	**1**	60.	**4**	75.	**2**

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SOLUTIONS

1. $(2, 3) \in R$ but $(3, 2) \notin R$.
Hence R is not symmetric.

2. $f(x) = {}^{7-x}P_{x-3}$
$7 - x \geq 0 \quad \Rightarrow \quad x \leq 7$
$x - 3 \geq 0 \quad \Rightarrow \quad x \geq 3$,
and $7 - x \geq x - 3 \quad \Rightarrow \quad x \leq 5$
$\Rightarrow 3 \leq x \leq 5 \Rightarrow x = 3, 4, 5 \Rightarrow$ Range is {1, 2, 3}.

3. Here $\omega = \frac{z}{i} \Rightarrow \arg\left(z.\frac{z}{i}\right) = \pi \Rightarrow 2\arg(z) - \arg(i) = \pi \Rightarrow \arg(z) = \frac{3\pi}{4}$.

4. $z = (p + iq)^3 = p\left(p^2 - 3q^2\right) - iq\left(q^2 - 3p^2\right)$

$$\Rightarrow \frac{x}{p} = p^2 - 3q^2 \ \& \ \frac{y}{q} = q^2 - 3p^2 \Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{\left(p^2 + q^2\right)} = -2.$$

5. $\left|z^2 - 1\right|^2 = \left(|z|^2 + 1\right)^2 \Rightarrow \left(z^2 - 1\right)\left(\bar{z}^2 - 1\right) = |z|^4 + 2|z|^2 + 1$
$\Rightarrow z^2 + \bar{z}^2 + 2z\bar{z} = 0 \Rightarrow z + \bar{z} = 0$
$\Rightarrow$ R (z) = 0 $\Rightarrow$ z lies on the imaginary axis.

6. $$A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

7. AB = I $\Rightarrow$ A(10 B) = 10 I

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha - 5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{if } \alpha = 5.$$

8. $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$C_3 \to C_3 - C_2$, $C_2 \to C_3 - C_1$

$$= \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0 \quad \text{(where r is a common ratio).}$$

9. Let numbers be a, b $\Rightarrow a + b = 18, \ \sqrt{ab} = 4 \quad \Rightarrow \quad ab = 16$, a and b are roots of the equation

$\Rightarrow \quad x^2 - 18x + 16 = 0$.

10. **(3)**

$(1-p)^2 + p(1-p) + (1-p) = 0$ (since (1 – p) is a root of the equation $x^2 + px + (1 - p) = 0$)

$\Rightarrow \quad (1-p)(1-p+p+1) = 0$

$\Rightarrow \quad 2(1-p) = 0 \Rightarrow (1 - p) = 0 \Rightarrow p = 1$

sum of root is $\alpha + \beta = -p$ and product $\alpha\beta = 1 - p = 0$ (where $\beta = 1 - p = 0$)

$\Rightarrow \quad \alpha + 0 = -1 \quad \Rightarrow \quad \alpha = -1 \Rightarrow$ Roots are 0, –1

11. $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$

S(k + 1)=1 + 3 + 5 +............ + (2k – 1) + (2k + 1)

= $(3 + k^2) + 2k + 1 = k^2 + 2k + 4$ [from $S(k) = 3 + k^2$]

= 3 + $(k^2 + 2k + 1)$ = 3 + $(k + 1)^2$ = S (k + 1).

Although S (k) in itself is not true but it considered true will always imply towards S (k + 1).

12. Since in half the arrangement A will be before E and other half E will be before A.

Hence total number of ways = $\frac{6!}{2}$ = 360.

13. Number of balls = 8

number of boxes = 3

Hence number of ways = ${}^7C_2$ = 21.

14. Since 4 is one of the root of $x^2 + px + 12 = 0 \Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

and equation $x^2 + px + q = 0$ has equal roots

$\Rightarrow D = 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$.

15. Coefficient of Middle term in $(1 + \alpha x)^4 = t_3 = {}^4C_2 \cdot \alpha^2$

Coefficient of Middle term in $(1 - \alpha x)^6 = t_4 = {}^6C_3 (-\alpha)^3$

${}^4C_2\alpha^2 = -{}^6C_3.\alpha^3 \Rightarrow \; -6 = 20\alpha \quad \Rightarrow \quad \alpha = \frac{-3}{10}$

16. Coefficient of x^n in $(1 + x)(1 - x)^n = (1 + x)({}^nC_0 - {}^nC_1x + \dots\dots + (-1)^{n-1}\, {}^nC_{n-1}\, x^{n-1} + (-1)^n$ ${}^nC_n\, x^n)$

= $(-1)^n\, {}^nC_n + (-1)^{n-1}\, {}^nC_{n-1} = (-1)^n (1 - n)$.

17. $t = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r} \quad (\because\ {}^nC_r = {}^nC_{n-r})$

$2t_n = \sum_{r=0}^{n} \frac{r+n-r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n}{{}^nC_r} \Rightarrow \; t_n = \frac{n}{2}\sum_{r=0}^{n} \frac{1}{{}^nC_r} = \frac{n}{2}S_n \; \Rightarrow \; \frac{t_n}{S_n} = \frac{n}{2}$

18. $T_m = \frac{1}{n} = a + (m-1)d$ .....(1)

and $T_n = \frac{1}{m} = a + (n-1)d$ .....(2)

from (1) and (2) we get $a = \frac{1}{mn}$, $d = \frac{1}{mn}$

Hence a – d = 0

19. If n is odd then (n – 1) is even ⇒ sum of odd terms $= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$.

20. $\frac{e^{\alpha} + e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots..$

$\frac{e^{\alpha} + e^{-\alpha}}{2} - 1 = \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots.$

put α = 1, we get

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\ldots.$

21. $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$.

Squaring and adding, we get

$2 + 2\cos(\alpha - \beta) = \frac{1170}{(65)^2}$

$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{130} \Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ $\left(\because \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}\right)$.

22. $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$= \sqrt{\frac{a^2+b^2}{2} + \frac{a^2-b^2}{2}\cos 2\theta} + \sqrt{\frac{a^2+b^2}{2} + \frac{b^2-a^2}{2}\cos 2\theta}$

$\Rightarrow u^2 = a^2 + b^2 + 2\sqrt{\left(\frac{a^2+b^2}{2}\right)^2 - \left(\frac{a^2-b^2}{2}\right)^2\cos^2 2\theta}$

min value of $u^2 = a^2 + b^2 + 2ab$

max value of $u^2 = 2\left(a^2 + b^2\right)$

$\Rightarrow u^2_{max} - u^2_{min} = (a-b)^2$.

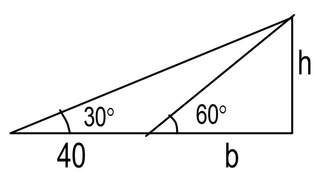
23. Greatest side is $\sqrt{1 + \sin\alpha\cos\alpha}$, by applying cos rule we get greatest angle = 120°.

24. $\tan 30° = \frac{h}{40 + b}$

$\Rightarrow \sqrt{3}h = 40 + b$ …..(1)

$\tan 60°$ = h/b ⇒ $h = \sqrt{3}$ b ….(2)

⇒ b = 20 m



25. $-2 \le \sin x - \sqrt{3}\cos x \le 2 \Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

⇒ range of f(x) is [–1, 3].

Hence S is [–1, 3].

26. If y = f (x) is symmetric about the line x = 2 then f(2 + x) = f(2 – x).

27. $9 - x^2 > 0$ and $-1 \le x - 3 \le 1 \Rightarrow x \in [2, 3)$

28. $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)\times 2x\times\left(\frac{a}{x}+\frac{b}{x^2}\right)} = e^{2a} \Rightarrow a = 1,\ b \in R$

29. $f(x) = \frac{1-\tan x}{4x-\pi} \Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi} = -\frac{1}{2}$

30. $x = e^{y+e^{y+e^{y+\dots\infty}}} \Rightarrow x = e^{y+x}$

$\Rightarrow \ln x - x = y \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$.

31. Any point be $\left(\frac{9}{2}t^2,\ 9t\right)$; differentiating $y^2 = 18x$

$\Rightarrow \frac{dy}{dx} = \frac{9}{y} = \frac{1}{t} = 2$ (given) $\Rightarrow t = \frac{1}{2}$.

$\Rightarrow$ Point is $\left(\frac{9}{8},\ \frac{9}{2}\right)$

32. $f''(x) = 6(x - 1) \Rightarrow f'(x) = 3(x - 1)^2 + c$
and $f'(2) = 3 \Rightarrow c = 0$
$\Rightarrow f(x) = (x - 1)^3 + k$ and $f(2) = 1 \Rightarrow k = 0$
$\Rightarrow f(x) = (x - 1)^3$.

33. Eliminating θ, we get $(x - a)^2 + y^2 = a^2$.
Hence normal always pass through (a, 0).

34. Let $f'(x) = ax^2 + bx + c \Rightarrow f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$

$\Rightarrow f(x) = \frac{1}{6}\left(2ax^3 + 3bx^2 + 6cx + 6d\right)$, Now f(1) = f(0) = d, then according to Rolle's theorem

$\Rightarrow f'(x) = ax^2 + bx + c = 0$ has at least one root in (0, 1)

35. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}} = \int_0^1 e^x dx = (e-1)$

36. Put $x - \alpha = t$

$\Rightarrow \int\frac{\sin(\alpha+t)}{\sin t}dt = \sin\alpha\int\cot t\,dt + \cos\alpha\int dt$

$= \cos\alpha(x-\alpha) + \sin\alpha\ln|\sin t| + c$

$A = \cos\alpha,\ B = \sin\alpha$

37. $\int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}}\int \frac{1}{\cos\left(x+\frac{\pi}{4}\right)}dx = \frac{1}{\sqrt{2}}\int \sec\left(x+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$

38. $\int_{-2}^{-1}\left(x^2-1\right)dx + \int_{-1}^{1}\left(1-x^2\right)dx + \int_{1}^{3}\left(x^2-1\right)dx = \left.\frac{x^3}{3}-x\right|_{-2}^{-1} + \left.x-\frac{x^3}{3}\right|_{-1}^{1} + \left.\frac{x^3}{3}-x\right|_{1}^{3} = \frac{28}{3}.$

39. $\int_{0}^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{\sqrt{(\sin x+\cos x)^2}}dx = \int_{0}^{\frac{\pi}{2}}(\sin x+\cos x)dx = \left|-\cos x+\sin x\right|_{0}^{\frac{\pi}{2}} = 2.$

40. Let $I = \int_0^{\pi} xf(\sin x)dx = \int_0^{\pi}(\pi - x)f(\sin x)dx = \pi\int_0^{\pi} f(\sin x)dx - I$ (since f (2a – x) = f (x))

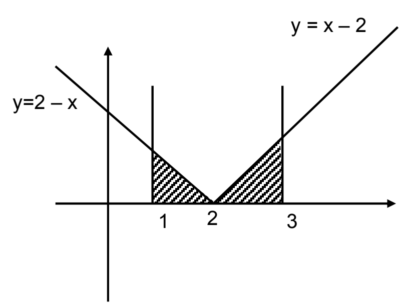
$\Rightarrow I = \pi\int_0^{\pi/2} f(\sin x)dx \Rightarrow A = \pi.$

41. f(-a) + f(a) = 1

$I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx = \int_{f(-a)}^{f(a)}(1-x)g\{x(1-x)\}dx$ $\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right)$

$2I_1 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx = I_2 \Rightarrow I_2 / I_1 = 2.$

42. Area $= \int_1^2 (2-x)dx + \int_2^3 (x-2)dx = 1.$



43. $2x + 2yy' - 2ay' = 0$

$a = \frac{x+yy'}{y'}$ (eliminating a)

$\Rightarrow (x^2 - y^2)y' = 2xy.$

45. $y\,dx + x\,dy + x^2y\,dy = 0.$

$\frac{d(xy)}{x^2y^2} + \frac{1}{y}dy = 0 \Rightarrow -\frac{1}{xy} + \log y = C.$

45. If C be (h, k) then centroid is (h/3, (k – 2)/3) it lies on 2x + 3y = 1.

$\Rightarrow$ locus is 2x + 3y = 9.

46. $\frac{x}{a}+\frac{y}{b}=1$ where a + b = -1 and $\frac{4}{a}+\frac{3}{b}=1$
$\Rightarrow$ a = 2, b = -3 or a = -2, b = 1.
Hence $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

47. $m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$
$m_1 + m_2 = 4m_1m_2$ (given)
$\Rightarrow$ c = 2.

48. $m_1 + m_2 = \frac{1}{4c}$, $m_1m_2 = \frac{6}{4c}$ and $m_1 = -\frac{3}{4}$.
Hence c = -3.

49. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow c = 4$ and it passes through (a, b)
$\Rightarrow a^2 + b^2 + 2ga + 2fb + 4 = 0$.
Hence locus of the centre is $2ax + 2by – (a^2 + b^2 + 4) = 0$.

50. Let the other end of diameter is (h, k) then equation of circle is
$(x – h)(x – p) + (y – k)(y – q) = 0$
Put y = 0, since x-axis touches the circle
$\Rightarrow x^2 – (h + p)x + (hp + kq) = 0 \Rightarrow (h + p)^2 = 4(hp + kq)$ (D = 0)
$\Rightarrow (x – p)^2 = 4qy$.

51. Intersection of given lines is the centre of the circle i.e. (1, – 1)
Circumference = $10\pi \Rightarrow$ radius r = 5
$\Rightarrow$ equation of circle is $x^2 + y^2 – 2x + 2y – 23 = 0$.

52. Points of intersection of line y = x with $x^2 + y^2 – 2x = 0$ are (0, 0) and (1, 1)
hence equation of circle having end points of diameter (0, 0) and (1, 1) is
$x^2 + y^2 – x – y = 0$.

53. Points of intersection of given parabolas are (0, 0) and (4a, 4a)
$\Rightarrow$ equation of line passing through these points is y = x
On comparing this line with the given line 2bx + 3cy + 4d = 0, we get
d = 0 and $2b + 3c = 0 \Rightarrow (2b + 3c)^2 + d^2 = 0$.

54. Equation of directrix is $x = a/e = 4 \Rightarrow a = 2$
$b^2 = a^2 (1 – e^2) \Rightarrow b^2 = 3$
Hence equation of ellipse is $3x^2 + 4y^2 = 12$.

55. $l = \cos\theta$, $m = \cos\theta$, $n = \cos\beta$
$\cos^2\theta + \cos^2\theta + \cos^2\beta = 1 \Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)
$\cos^2\theta = 3/5$.

56. Given planes are
$2x + y + 2z – 8 = 0$, $4x + 2y + 4z + 5 = 0 \Rightarrow 2x + y + 2z + 5/2 = 0$
Distance between planes = $\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}} = \frac{|-8-5/2|}{\sqrt{2^2+1^2+2^2}} = \frac{7}{2}$.

57. Any point on the line $\frac{x}{1}=\frac{y+a}{1}=\frac{z}{1}=t_1$ (say) is $(t_1, t_1 - a, t_1)$ and any point on the line $\frac{x+a}{2}=\frac{y}{1}=\frac{z}{1}=t_2$ (say) is $(2t_2 - a, t_2, t_2)$.
Now direction cosine of the lines intersecting the above lines is proportional to $(2t_2 - a - t_1, t_2 - t_1 + a, t_2 - t_1)$.
Hence $2t_2 - a - t_1 = 2k$, $t_2 - t_1 + a = k$ and $t_2 - t_1 = 2k$
On solving these, we get $t_1 = 3a$, $t_2 = a$.
Hence points are (3a, 2a, 3a) and (a, a, a).

58. Given lines $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$ and $\frac{x}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$ are coplanar then plan passing through these lines has normal perpendicular to these lines
$\Rightarrow a - b\lambda + c\lambda = 0$ and $\frac{a}{2}+b-c=0$ (where a, b, c are direction ratios of the normal to the plan)
On solving, we get $\lambda = -2$.

59. Required plane is $S_1 - S_2 = 0$
where $S_1 = x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
$S_2 = x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
$\Rightarrow 2x - y - z = 1$.

60. $\left(\vec{a}+2\vec{b}\right)=t_1\vec{c}$ ....(1)
and $\vec{b}+3\vec{c}=t_2\vec{a}$ ....(2)
$(1) - 2\times(2) \Rightarrow \vec{a}\left(1+2t_2\right)+\vec{c}\left(-t_1-6\right)=0 \Rightarrow 1+2t_2 = 0 \Rightarrow t_2 = -1/2$ & $t_1 = -6$.
Since $\vec{a}$ and $\vec{c}$ are non-collinear.
Putting the value of t_1 and t_2 in (1) and (2), we get $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$.

61. Work done by the forces $\vec{F}_1$ and $\vec{F}_2$ is $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$, where $\vec{d}$ is displacement
According to question $\vec{F}_1+\vec{F}_2 = (4\hat{i}+\hat{j}-3\hat{k})+(3\hat{i}+\hat{j}-\hat{k})=7\hat{i}+2\hat{j}-4\hat{k}$
and $\vec{d}=(5\hat{i}+4\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=4\hat{i}+2\hat{j}-2\hat{k}$. Hence $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$ is 40.

63. Condition for given three vectors to be coplanar is $\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix} = 0 \Rightarrow \lambda = 0, 1/2$.
Hence given vectors will be non coplanar for all real values of λ except 0, 1/2.

63. Projection of $\bar{v}$ along $\bar{u}$ and $\bar{w}$ along $\bar{u}$ is $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}$ and $\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|}$ respectively
According to question $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}=\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|} \Rightarrow \bar{v}\cdot\bar{u}=\bar{w}\cdot\bar{u}$. and $\bar{v}\cdot\bar{w}=0$
$|\bar{u}-\bar{v}+\bar{w}|^2 = |\bar{u}|^2 + |\bar{v}|^2 + |\bar{w}|^2 - 2\bar{u}\cdot\bar{v}+2\bar{u}\cdot\bar{w}-2\bar{v}\cdot\bar{w} = 14 \Rightarrow |\bar{u}-\bar{v}+\bar{w}| = \sqrt{14}$.

64. $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a} \Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{b}\cdot\vec{c}\right)\vec{a} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$

$\Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} = \left(\frac{1}{3}|\bar{b}||\bar{c}| + \left(\bar{b}\cdot\bar{c}\right)\right)\vec{a} \Rightarrow \vec{a}\cdot\vec{c} = 0$ and $\frac{1}{3}|\bar{b}||\bar{c}| + \left(\bar{b}\cdot\bar{c}\right) = 0$

$\Rightarrow |\bar{b}||\bar{c}|\left(\frac{1}{3} + \cos\theta\right) = 0 \Rightarrow \cos\theta = -1/3 \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$.

65. Mode can be computed from histogram and median is dependent on the scale.
Hence statement (a) and (b) are correct.

66. $x_i = a$ for $i = 1, 2, \ldots, n$ and $x_i = -a$ for $i = n, \ldots, 2n$

S.D. = $\sqrt{\frac{1}{2n}\sum_{i=1}^{2n}(x_i - \bar{x})^2} \Rightarrow 2 = \sqrt{\frac{1}{2n}\sum_{i=1}^{2n}x_i^2} \quad \left(\text{Since } \sum_{i=1}^{2n} x_i = 0\right) \Rightarrow 2 = \sqrt{\frac{1}{2n}\cdot 2na^2} \Rightarrow |a| = 2$

67. E_1 : event denoting that A speaks truth
E_2 : event denoting that B speaks truth

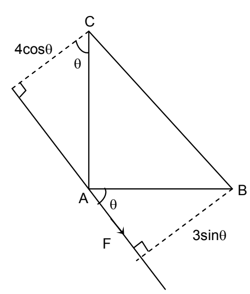
Probability that both contradicts each other = $P\left(E_1 \cap \bar{E}_2\right) + P\left(\bar{E}_1 \cap E_2\right) = \frac{4}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{3}{4} = \frac{7}{20}$

68. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$

69. Given that n p = 4, n p q = 2 $\Rightarrow$ q = 1/2 $\Rightarrow$ p = 1/2 , n = 8 $\Rightarrow$ p(x = 2) = ${}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = \frac{28}{256}$

70. P + Q = 4, $P^2 + Q^2 = 9 \Rightarrow P = \left(2 + \frac{1}{2}\sqrt{2}\right)$N and $Q = \left(2 - \frac{1}{2}\sqrt{2}\right)$N.

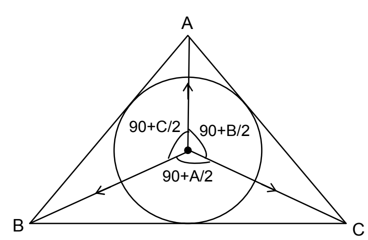
71. F . 3 sin θ = 9
F . 4 cos θ = 16
$\Rightarrow$ F = 5.



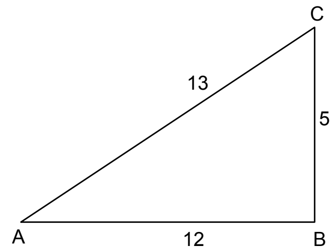
72. By Lami's theorem

$\vec{P} : \vec{Q} : \vec{R} = \sin\left(90° + \frac{A}{2}\right) : \sin\left(90° + \frac{B}{2}\right) : \sin\left(90° + \frac{C}{2}\right)$

$\Rightarrow \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$.



73. Time T_1 from A to B = $\frac{12}{4}$ = 3 hrs.



T_2 from B to C = $\frac{5}{5}$ = 1 hrs.

Total time = 4 hrs.

Average speed = $\frac{17}{4}$ km/ hr.

Resultant average velocity = $\frac{13}{4}$ km/hr.

74. Component along OB = $\frac{\frac{1}{4}\sin 30°}{\sin(45° + 30°)} = \frac{1}{8}\left(\sqrt{6} - \sqrt{2}\right)$ m/s.

75. $t_1 = \frac{2u\sin\alpha}{g}$, $t_2 = \frac{2u\sin\beta}{g}$ where $\alpha + \beta = 90^0$

$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$.