IPMAT INDORE 2022 Answer Key and Explanations

(344) It is given that the 3rd, 14th and 69th terms of given AP is forming three distinct and consecutive terms of a G.P. 1. By observation, we can say that position 3rd, 14th and 69th follow a certain sequence i.e., $(3 \times 5) - 1 = 14$ $(14 \times 5) - 1 = 69$ Similarly, (69 × 5)-1 = 345-1= 344 So, next term in G.P. will be 344th term of the given AP. **Alternate Method** $G_1 = A_3 = a + 2d,$ $G_2 = A_{14} = a + 13d$, and $G_3 = A_{69} = a + 68d$ are in G.P. We know that, if any three terms a, b, c are in G.P., then common ratio, $r = \frac{b}{a} = \frac{c}{b}$ or $ac = b^2$ Applying the same, $(a + 2d) (a + 68d) = (a + 13d)^2$ \Rightarrow a²+68ad +2ad +136d² = a²+26d+169d² ⇒ $44ad = 33d^2$ $\frac{a}{d} = \frac{3}{4}$. Now let a = 3x and d = 4x⇔ Common ratio, $r = \frac{a+13d}{a+2d}$ = $\frac{3x + 13 \times 4x}{3x + 8x} = \frac{55x}{11x} = 5$ Next term of GP will be $G_4 = G_3 \times r = (a + 68d) \times 5$ $= (3x + 272x) \times 5$ $= 275x \times 5$ = 1375 xLet G_4 be the nth term of given AP. Therefore, a + (n - 1)d = 1375x3x + (n - 1)4x = 1375xn = 344. 2. (12) $2|X| + 3|Y| \le 6$ Above inequality represent an area bounded by 4 lines which are (i) 2x + 3y = 6(ii) 2x - 3y = 6(iii) -2x + 3y = 6(iv) -2x - 3y = 6(0, 2) (3, 0 (0, -2) By drawing these 4 lines on graph, we get a rhombus. Area of rhombus = $\frac{1}{2}$ d₁. d₂ $=\frac{1}{2} \times 4 \times 6$ = 12 sq. units 3. (175)(7 points) (5 point) Case I – Choosing 2 points from L₁ and 1 point form $L_2 = {^7C_2 \times {^5C_1}}$ Case – II – Choosing 2 points form L_2 and 1 point form $L_1 = {}^{5}C_2 \times {}^{7}C_1$ Total case = $({}^{7}C_{2} \times {}^{5}C_{1}) + ({}^{5}C_{2} \times {}^{7}C_{1})$ $= 35 \times 5 + 10 \times 7$



= 105 + 70 = 175(100) Given sina + sin $\beta = \frac{\sqrt{2}}{\sqrt{3}}$ 4. Squaring both sides, we get $(\sin \alpha + \sin \beta)^2 = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2$ $=\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = \frac{2}{3}$ -----(1) & $\cos\alpha + \cos\beta = \frac{1}{\sqrt{3}}$ $(\cos\alpha + \cos\beta)^2 = \left(\frac{1}{\sqrt{3}}\right)^2$ $=\cos^2\alpha + \cos^2\beta + 2\cos\alpha.\cos\beta = \frac{1}{3} \quad -----(2)$ Adding equations (1) and (2), we get $\sin^2 \alpha + \sin^2 \beta + 2\sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2\cos \alpha \cdot \cos \beta = 1$ \Rightarrow 1 + 1 + 2 [sina.sin β + cosa.cos β] = 1 $\Rightarrow 2\cos(\alpha - \beta) = -1$ $\Rightarrow \cos(\alpha - \beta) = \frac{-1}{2}$ $\Rightarrow 2\cos^2\left(\frac{\alpha-\beta}{2}\right) \cdot 1 = \frac{-1}{2} [::\cos 2A = \cos^2 A \cdot 1]$ $\Rightarrow 2\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$ $\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}$ We have to find the values of $\left(20 \cos\left(\frac{\alpha-\beta}{2}\right)\right)^2$ $\left(20\cos\left(\frac{\alpha-\beta}{2}\right)\right)^2 = (20)^2\cos^2\left(\frac{\alpha-\beta}{2}\right)$ $=400 \times \frac{1}{4}$ = 100.5. (256) Using Binomial Theorem, $(5x - 9)^4 = {}^4C_0(5x)^4(-9)^0 + {}^44_1(5x)^3(-9)^1 + \dots + {}^4(4(5x)^0(-9)^4)^4$

We can obtain sum of coefficients of all the terms in the expansion by substituting x =1 on the RHS. As RHS equals to LHS, we can get the sum of coefficients even by substituting x = 1 in LHS only. \therefore Put x = 1 in L.H.S expression, we get $(5.1-9)^4 = (-4)^4 = 256$

6. **(50)**

7.

	Unit Price of Apples	Quantity	Unit Price of Mangoes (Rs. /kg.)	Quantity
	(Rs. /kg.)	(in kg.)		(in kg.)
CP	20	A	25	М
SP (1)	22	A	20	М
SP (2)	16	A	27.5	M

In first case, there is no profit or loss, means Total SP = Total CP 22A + 20M = 20A + 25MA:M=5:2 Let A = 5x & M = 2xIn second there is a loss of Rs.150 Total CP - Total SP = Rs.150 20A + 25M - (16A +27.5M) = Rs.150 4A-2.5M=150 20x-5x=150x = Rs.10Number of apples purchased, A = 5x = 50. (72) Given $\log_{x^2} y + \log_{y^2} x = 1$ and $y = x^2 - 30$ $\log_{x^2} y + \log_{y^2} x = 1$ $=\frac{1}{2}[\log_{x} y + \log_{y} x] = 1$ $= \log_x y + \log_y x = 2$

$$= \log_{x} y + \frac{1}{\log_{x} y} = 2 \left[\therefore \log_{y} x = \frac{1}{\log_{y} x} \right] - \dots - eqn.(1)$$

We know, if a is a positive number, then a $+\frac{1}{a} = 2$, only when a = 1

Applying it in eq. (1), we can say

 $\log_x y = 1$ or y=xSo putting y = x, in the 2nd given eqn. $y = x^2 - 30$ we get $x = x^2 - 30$

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 $x^{2} - x - 30 = 0$ $x^{2} + 5x - 6x - 30 = 0$ x (x + 5) - 6 (x + 5) = 0 (x + 5) (x - 6) = 0 X = -5 or 6As x cannot be negative number, $\therefore x = y = 6$ $\therefore x^{2} + y^{2} = 6^{2} + 6^{2}$ = 36 + 36= 72

8. **(16)** Power set of A set is set of all the subsets of given set.

If a set contains n elements, total no. of sub-sets will be equal 2ⁿ.

Set	No of elements	No. of sub-sets (2 ⁿ)
A	0	2°=1=No. elements in P(A)
P(A)	1	$2^1=2=$ No. of elements in P(P(A))
P(P(A))	2	$2^2=4=$ No. of elements in P(P(P(A))
P(P(P(A)))	4	$2^4=16=$ No. of elements in P(P(P(P(A))))

9. (2067)

Position from left end	1	2	3	4	5	6	7	8	9		100		2022	 2025
Old series	1	2	3	4	5	6	7	8	9	-	100	- 1	2022	 2025
Perfect Square	12			22		 			3 ²		10 ²			 45 ²
New series	2=1+1	3	5	6 = 4+ 2	7	8	9	10	12=9 +3				2067	2070 = 2025+45

So, in the new series the number at 2025^{th} position will be 2025 + 45 = 2070 \therefore The number at 2022^{nd} position will be 2067.

10. (32) Given
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

 \therefore We can say that
 $A^{6} = A^{2}. A^{2} A^{2} = I.I.I = I$
 $A^{3} = A^{2}. A = I.A = A$
 $A^{9} = A^{3}.A^{3}.A^{3} = A.A.A = A^{2}.A = I.A = A$
 $A^{9} + A^{6} + A^{3} + A = A + I + A + A$
 $= 3A + I$
 $= 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
 $a \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Determinant of above matrix = 4(1×1-3×3)
 $= 4(1-9) = 4(-8)$
 $= -32$
Absolute value of $-32 = |-32| = 32$.
11. (3) If a, b, c are 3 terms in A.P
then $\frac{a+b}{2} = b$ or, $a+c = 2b$
So, $-16, 2^{x+3} - 2^{2x-1} - 16, 2^{2x-1} + 16$ are in A.P.
We can say, $-16 + 2^{2x-1} + 16 = 2[2^{x+3} - 2^{2x-1} - 16]$
 $\Rightarrow 2^{2x} 2^{-1} = 2^{x+4} - 2^{2x} - 32 - \dots - eqn.(1)$



put $2^x = t \& 2^{2x} = t^2$ the eqn. becomes $\frac{t^2}{2} = 16t - t^2 - 32$ \Rightarrow t²= 32t - 2t²-32 × 2 ⇔ $3t^2 - 32t + 32 \times 2 = 0$ $\Rightarrow \quad 3t^2 - 32t + 64 = 0$ \Rightarrow t = 8 or $\frac{8}{3}$ As $t = 2^x$, it can not be a fraction \therefore t = 2^x = 8 (only) $\therefore x = 3$ 12. (26) $F(x) = |x| + 2|x - 1| + |x - 2| + |x - 4| + |x - 6| + 2|x - 10|, x \in (-\infty, \infty)$ Minimum value of f(x) will be obtained at one of the six critical points, which are x = 0, 1, 2, 4, 6, 10. So, f(0) = 2 + 2 + 4 + 6 + 20 = 30f(1) = 1 + 1 + 3 + 5 + 18 = 28f(2) = 2 + 2 + 2 + 4 + 16 = 26f(4) = 4 + 6 + 2 + 0 + 2 + 12 = 26f(6) = 6 + 10 + 4 + 2 + 8 = 30f(10) = 10 + 18 + 8 + 6 + 4 = 46Thus, minimum value f(x) = 26Sol. Let the distance between his home and office is 'd' km 13. (30) Usual time taken $= \frac{d}{\frac{12}{d}}$ km New time taken = $\frac{a}{20}$ km As per the question, difference between these two times is I hour. $\frac{d}{12} - \frac{d}{20} = 1$ $d\left(\frac{5-3}{60}\right) = 1$ $d = \frac{60}{2} = 30$ = 30 km(3150) We have to find maximum value of the sum of 50 numbers. 14. Highest of all the distinct 50 numbers, is 100. So, in order to maximum the sum of all 50 numbers, highest 25 numbers should be 76, 77, 78,.....,100. There average of these 25 numbers will be 88.

According to the question, rest of the 25 numbers should have their average 88 - 50 = 38

Thus, the sum of those 25 numbers will be 38x25.

Sum of all these 50 nos. = 25 × 88 + 25 × 38 = 25 × 126

= 3150

15. (960)Number of ways in which Mr. and Mrs Sharma are always seated together



= 6!×2 = 720×2 = 1440 ways





Number of ways in which Mr. & Mrs. Sharma as were as Mr. & Mrs. Ahuja both couples are seated such that their spouse is next to each other.

 $=5! \times 2 \times 2 = 480$

 \therefore The total no. of ways in which Mr. and Mrs Sharma are seated next to each other but Mr Ahuja and Mrs. Ahuja does not sit next to each other

= 1440 - 480 = 96016. (a) $x^2 + |x+4| + |x-4| - 35 = 0$ Case I: if $x \ge 4$ Then, $x^2 + (x + 4) + (x - 4) - 35 = 0$ $x^2 + 2x - 35 = 0$ $x^2 + 7x - 5x - 35 = 0$ x(x+7) - 5(x+7) = 0(x-5)(x+7)=0x = 5 or x = -7Only x=5 satisfies the assumption. Case II: If x < -4. Then $x^2 - (x + 4) - (x - 4) - 35 = 0$. $x^2 - 2x - 35 = 0$ $x^2 + 5a - 7x - 35 = 0$ x(x+5) - 7(x+5) = 0(x-7)(x+5) = 0X = 7 or x = -5Only x = -5 satisfies the assumptions. Case III: If $-4 < x \le 4$ Then, $x^2 + (x + 4) - (x - 4) - 35 = 0$ $x^2 + x + 4 - x + 4 - 35 = 0$ $x^2 - 27 = 0$ $x = \sqrt[4]{27}$ or $x = \sqrt{27}$ None of these values of 'x' satisfies the assumption. ∴ only 2 roots i.e. 5 and –5 will be there Their sum = $5^2 + (-5)^2 = 25 + 25 = 50$ 17. (c) If Set $A = \{1, 2, 3 \dots m' \text{ elements}\}$ and Set $B = \{a, b, c, \dots, n' \text{ elements}\}$ Then total number of relations = 2^{mn} & Total no. of all possible functions = n^m (From set A to set B) & Total number of all possible functions = m^n (From set B to set A) As per the question, A= [1,2,3] = 3 elements. $B = \{a, b\} = 2$ elements. : Total number of functions from set A to set $B = 2^3$ & Total number of relations $=2^{2\times 3}=2^6$ $\therefore \mathbf{P}(E) = \frac{2^3}{2^6} = \frac{1}{2^3} = \frac{1}{8}.$ (a) Given abcde is divisible by 6 18. Means, abcde is divisible by both 2 & 3. It mean (a+b+c+d+e) is always divisible by 3 (using divisibility rule of 3) And the value of 'e' is certainly 0, 2, 4, 6 or 8. Now going by options, Options (b) eee \Rightarrow this no. be divisibly by 3 & 2 both (means by 6 as well) because sum of digit is 3e & unit digit is same as that of original number.



Option (c)

bbadcacede \Rightarrow again the sum of digit = 2 (b+a+d+e+c) & unit digit is again 'e' same as that of the original no. Options (d) cdbae is also divisible by 3 and 2 both (thus by 6) for the same reasons. Unit digit in above 3 options is 'e' which is same as that of the original number. Option1. Unit digit is 'a' and we are not sure whether it is even or odd, so edcba is not necessarily divisible by 2 & thus not by 6.

(a) $\log_{27} 8 \le \log_3 x < 9^{10}$ 19.

 $\log_{3^{3}} 8 \le \log_{3} x < 9^{\frac{1}{\log_{2} 3}} \left[\text{Using } \log_{m^{n}} a = \frac{1}{n} \cdot \log_{m} a \& \frac{1}{\log_{b} a} = \log_{a} b \right]$ $\frac{1}{3} \cdot \log_{3} 8 \le \log_{3} x < 3^{2 \cdot \log_{3} 2}$ $\log_{3}(8)^{\frac{1}{3}} \le \log_{3} x < 3^{\log_{3} 4}$ $\log_{3} 2 \le \log_{3} x < 4$ $\log_{3} 2 \le \log_{3} x < 4 \cdot \log_{3} 3$ $4 \cdot \log_{3} 3$ [Using $a^{\log_{a} x} = x$] $\log_3 2 \le \log_3 x < \log_3(3)^4$ $\log_3 2 \le \log_3 x < \log_3 81$ $\Rightarrow 2 \leq x < 81 \text{ or } [2, 81)$

(c) Suppose a, b and c are integers such that a > b > c > 0, and $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ Then the value of the determinant of A 20.

Determinant Δ = a (bc-a²) – b (b² – ac) + c (ab-c²) $= 3 abc - a^3 - b^3 - c^3$ Assuming a = 3, b = 2 and c = 1 (As given that a > b > c > 0) $\Delta = (3 \times 3 \times 2 \times 1) - 3^3 - 2^3 - 1^3 = 18 - 38 = -20$, which is negative So, Δ will always be negative.

(c) Three sides of the triangle are x, 21 and 40, where the shorter side is x. 21. We know, in a triangle the sum of any two sides is always greater than the third side. Using it, we get x + 21 > 40Or, x > 40 - 21As x > 19

 \therefore Possible value of x will be 20.

22. (b) Note - Above question had typo error in IPMAT Indore-2022. Objection was raised by the students, which was accepted by IIM authorities and bonus marks were awarded to all the students who attempted this question. We have amended the question to provide the solution.

Amended Question:

Similarly, $A^n = A$ $\therefore A^{2022} = A$

(b) Sum of first 'n' term of an AP, 23.

> $S_n = \frac{n}{2} [2a + (n-1)d]$ Sum of first 15 terms = $\frac{15}{2}$ [2a + 14d] = 200 \Rightarrow a + 7d = $\frac{40}{3}$ -----(I) Sum of next 15 terms = Sum of first 30 terms – Sum of first 15 terms $=\frac{30}{2}[2a+29d]-\frac{15}{2}[2a+14d]=350$ $\Rightarrow 4a + 58d - 2a - 14d = \frac{140}{3}$ $\Rightarrow 2a + 44d = \frac{140}{3} \Rightarrow a + 32d = \frac{70}{3} - \dots (2)$

Solving equation (1) and equation (2), we get $d = \frac{2}{3}$

(a) $Cost \propto (Weight)^2$ (given) $C \propto w^2$

24.

 $\frac{C_1}{C_2} = \frac{W_1^2}{W_2^2}$ $\frac{3600}{C_2} = \frac{10^2}{4^2}$ $=\left(\frac{10}{4}\right)^2 = \left(\frac{5}{2}\right)^2$ $\frac{3600}{C_2} = \frac{25}{4}$



$$C_{2} = 560 \times \frac{2}{25}$$

$$= 36 \times 16 = 575$$

$$C_{2} = 576.$$
(2) Let the two numbers be a and b.
(3) Let the two numbers be a and b.
(4) Let the two numbers be a and b.
(5) $\frac{(a + b)^{2} - (a - b)^{2}}{4}$

$$= \frac{a^{2} + 2ab + b^{2} - (a^{2} - 2ab + b^{2})}{4}$$

$$= \frac{a^{2} + 2ab + b^{2} - (a^{2} - 2ab + b^{2})}{4}$$

$$= \frac{4ab}{4} = a \cdot b = Product of the numbers.$$
We also know,
Product of 2 number is always equal to product of their HCF and LCM.
(5) Let the speed of A shok, Bipin and Chandan be a, b and c m/s.
As per the question.
Bipin can cover 150m in 15 sec.
 \therefore His speed, b = $\frac{150}{16} = 10 \text{ m/s}.$
Also, difference in the time taken by Bipin and Ashok to reach the finishing line is 15 sec.
 $\therefore 400 - \frac{400}{10} - \frac{40}{40} = 15$
 $\therefore a - 15m/s$
Also $\frac{600}{c} - \frac{400}{16} = 25$
 $\therefore c - 8 \text{ m/s}.$
Now, the ratio of distance travelled by Bipin and Chandan will be in the ratio of their speeds, as the distance travelled by
them is same.
 $s_{0} \frac{400}{c} = \frac{10}{16} = x = 320$
Therefore, Bipin will be the Chandan by 80m.
27. (a) Bit + 817 = 3
Or, Bit = 3 - 817
Now, Bit will always be a positive number.
Means Bit > 0
 $\therefore 3 - 817 > 0$
 $\Rightarrow 3 - 817 = 0$
 $\Rightarrow 1 - 4y < 1$
Or $y < \frac{1}{2}$
 $\therefore (-m \frac{1}{4}) or, (-m 0.25)$
28. (b) It is given that the cartesian product of A × B consists of 4 elements, the set A and B can have 2 elements each.
Looking into the given 2 elements (1, 4) & (4, 1), we can determine the two sets.
 $A = (1.4)(4, (0.4), (1.4), (1.4))$
 $A × B = B \times A$

29. **(d)**



Given AD : DB = AE : EC = 1:1. Using Thale's theorem, we can say that the line DE will be parallel to BC. It means \triangle ADE & \triangle ABC are two right angled similar triangles.



Answer for the two similar triangles, the ratio of corresponding sides is equal.

 $\therefore \frac{BC}{DE} = \frac{AB}{AD}$ $\frac{BC}{6} = \frac{2x}{x} \Rightarrow BC = 12 \text{ cm}$

(a) After adding 100 ml. of apple juice to the bowl containing 60 ml. orange juice & 40 ml water, 30.

% orange juice in 200 ml fruit punch = $\frac{60}{200} \times 100 = 30\%$

(Madhu drinks 50 ml. from this 200 ml fruit punch. This will not change the percentage of orange juice in the remaining 150 ml fruit punch. It will remain 30%)

Let say in order to increase the taste Madhu adds 'x' ml of pure orange juice (100%) in this 150 ml of remaining fruit punch so as to make percentage of orange juice equal to 50%.

Applying the rule of alligation, we get



The ratio in which remaining fruit punch & pure orange juice should be mixed is 50 : 20 or 5 : 2

 $\therefore \frac{150}{x} = \frac{5}{2} \Rightarrow x = 60 \text{ ml.}$

(c) Average = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

Sum of heights of n persons = Average height × Number of persons $= 160 \times n.$

Similarly, sum of heights of 'm' persons = $172 \times m$ Average height of all persons

Sum of observaitons

Sum of height all the persons _ Total number of persons $160n \pm 172$ m

$$164 = \frac{100n + 172 \text{ m}}{\text{m} + \text{n}}$$

$$\Rightarrow 164(m + n) = 160n + 172m$$

$$= 4n = 8m$$

$$\Rightarrow \frac{m}{2} = \frac{4}{2} = \frac{1}{2}$$

$$m : n = 1:2$$

(b) Given $0 < \theta < \frac{\pi}{4}$, or $0 < \theta < 45^{\circ}$ 32.

 $a = \sin \theta^{\sin \theta} \log_2 \cos \theta$

 $b = \cos \theta^{\sin \theta} \log_2 \sin \theta$ $c = sin\theta^{cos \theta} log_2 cos \theta$

 $d = \sin \theta^{\sin \theta} \log_2 \sin \theta$

In order to find median of a, b, c, d, we need to arrange them in increasing or decreasing order. Then the mean value of middle two numbers will be the median of all four values.

Let us take
$$\theta = 30^{\circ}$$

Then $\sin 30^{\circ} = \frac{1}{2} = 0.5$
 $\cos 30^{\circ} = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866 = 0.87$
 $\log_2 \sin 30^{\circ} = \log_2 \frac{1}{2} = \log_2 1 - \log_2 2 = 0 - 1 = -1$
 $\log_2 \cos 30^{\circ} = \log_2 \frac{\sqrt{3}}{2} = \log_2 \sqrt{3} - \log_2 2 = \frac{1}{2} \log_2 3 - 1 = \frac{1}{2} \times 1.5 - 1$
 $= 0.75 - 1$
 $= 0.75 - 1$
 $= 0.25$
(We Assume $\log_2 2 < \log_2 3 < \log_2 4$)
(Or, $1 < \log_2 3 < 2$)
So, the 4 numbers comes out as,
 $a = 0.5^{0.5} \log_2 \cos 30^{\circ} = 0.5^{0.5} (-0.25)$
 $b = 0.87^{0.5} \log_2 \sin 30^{\circ} = 0.87^{0.5} (-1)$
 $c = 0.5^{0.87} \log_2 \cos 30^{\circ} = 0.5^{0.87} \times (-0.25)$
 $d = 0.5^{0.5} \log_2 \sin 30^{\circ} = 0.5^{0.5} \times (-1)$
All the numbers are negative.
So, the number with greatest magnitude (value) will be the least and the number with least magnitude (value) will be highest.



e.g. - 100 < - 2 Now comparing 'a' and 'd' clearly [a >d] Now comparing 'a' and 'c', We know, $0.5^{0.87} < 0.5^{0.5}$ $\therefore 0.5^{0.87} \times (-0.25) > 0.5^{0.5} \times (-0.25)$ ∴ c > a Now comparing 'd' and 'b', we get We know $0.87^{0.5} > 0.5^{0.5}$ $0.87^{0.5} \times (-1) < 0.5^{0.5} \times (-1)$ b < d or, d > bCombining all inequalities, we get c > a > d > bThus, median $= \frac{a+d}{2}$ (a) If $f(x^2 + f(y)) = xf(x) + y$ for all non-negative integers x and y, then the value of $[f(0)]^2 + f(0)$ equals_ 33. Sol. Given $f(x^2 + f(y)) = xf(x) + y$ Put x = 1 and y = 0 we get, $f(1 + f(0)) = 1 \times f(1) + 0$ \Rightarrow f(1 + f(0)) = f(1) It means 1+f(0) = 1 or f(0) = 0Therefore, $[f(0)]^2 + f(0) = 0$. (c) Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 34. & Equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ The equation of the given curve is $\frac{x^2}{\sin\sqrt{2}-\sin\sqrt{3}} + \frac{y^2}{\cos\sqrt{2}-\cos\sqrt{3}} = 1$ We know, foci always lies on major axis Also, If $a > b \Rightarrow x - axis$ will be the major axis & If $b > a \Rightarrow y - axis$ will be the major axis in both ellipse & hyperbola. Here, we need to figure out which expression is greater between $\sin\sqrt{2} - \sin\sqrt{3}$ and $\cos\sqrt{2} - \cos\sqrt{3}$. Note: Here $\sqrt{2}$ And $\sqrt{3}$ are the angles in radian and not degree. Also, we know $\sqrt{2} = 1.414$ $\frac{\pi}{2} = \frac{3.14}{2} = 1.57$ $\sqrt{3} = 1.732$ axis Sin x Cos x Sin x & Cos x x - axis 1.732 14 π 0 $\pi/2 = 1.57$ 1.414 Angle (in radian) -1 From the graph, it is visible that, $\sin\sqrt{2} > \sin\sqrt{3}$ $\therefore \sin \sqrt{2} - \sin \sqrt{3}$ is always positive, but it will have a very small value.

(Because, 1.732 is farther from 1.57 compared to 1.414, which makes sin1.732 lesser than sin 1.414.)

Also from the graph, we can see that $\cos\sqrt{3}$ will be a negative value.

It means, $\cos\sqrt{2} - \cos\sqrt{3}$ will be always be a positive value & will have larger magnitude compared to $\sin\sqrt{2} - \sin\sqrt{3}$. Thus, we can say that major axis, of given curve is y – axis & foci will be on this only.

35. **(d)** Number of 4-digit numbers divisible by both 2 and 3

= Number of 4-digit numbers divisible by the LCM (2 & 3) i.e. 6.

 $=\left[\frac{9999}{6}\right] - \left[\frac{1000}{6}\right] = 1666 - 166 = 1500$



Number of 4-digit numbers divisible by 2,3 and 5

- = Number of 4-digit numbers divisible by LCM (2,3,5) = i.e. 30.
- $= \left[\frac{9999}{30}\right] \left[\frac{1000}{30}\right] = 333 33 = 300$
- \therefore Number of 4-digit numbers divisibly by 2 & 3 but not 5
- = (Number of 4 -digit divisibly by 2 & 3 i.e. 6) (Number of 4-digit nos. divisibly by 2,3 &5)
- = 1500 300 = 1200.

Thus, speed of car is 4.39 km/h.

(b) Let the number is $N = 3^7 \cdot 2^8 \cdot 17^3$ Total number of factors of N = (7+1)(8+1)(3+1) $= 8 \times 9 \times 4 = 8 \times 36 = 288$

In order to find total number of perfect square factors, we need to find the number of perfect square factors of 3⁷, 2⁸ & 17³, which when multiplied in combinations will produce all possible perfect square factors. /2º\

0

$$\Rightarrow \begin{pmatrix} 3^{\circ} \\ 3^{2} \\ 3^{4} \\ 3^{6} \end{pmatrix} \begin{pmatrix} 2^{2} \\ 2^{4} \\ 2^{6} \\ 2^{8} \end{pmatrix} \times \begin{pmatrix} 17^{0} \\ 17^{2} \end{pmatrix}$$
$$= 4 \times 5 \times 2 = 40$$
$$\Rightarrow P(E) = \frac{40}{288} = \frac{5}{36}$$

38.

37.

(c) It is given that the equations represent 3 straight lines. Concurrent lines pass through a single point. So let us find the coordinates of the point from which first two lines pass. It can be obtained by solving, equations of first two lines. x - y - 1 = 0 (1) x = 2

$$\begin{array}{l} x - y - 1 = 0 \dots (1) \times 2 \\ \hline 2x - 3y + 2 = 0 \dots (2) \times 1 \\ \hline 2x - 2y - 2 = 0 \dots (3) \\ \hline 2x + 3y - 12 = 0 \dots (4) \\ \hline \text{solving equations (3) and (4) we let} \\ y = 2; x = 3 \\ \hline \text{So, the point (3, 2) should also satisfy the questions of third line, i.e., } 2x - 3y + k = \\ \hline \text{Putting the values of } x = 3 \& y = 2 \text{ in it, we put} \\ 2 \times (3) - 3(2) + k = 0 \\ \hline 6 - 6 + k = 0 \\ \therefore k = 0. \end{array}$$



39. (a) $f(x) = \frac{x}{|x|}$

Case I: If $x \ge 0$, $|x| = +x \therefore f(x) = \frac{x}{-x} = 1$ Case II: if x < 0, $|x| = -x \therefore f(x) = \frac{x}{-x} = -1$ \therefore Solution set will be $\{1, -1\}$.

40. (c) MANAGEMENT.

This word has 4 vowels (2A's and 2E's). and 6 consonants (2M's, 2N's one G & one T). _ M _ N_G_M_N_T_

We will arrange all the consonants first.

It can be arranged in $\frac{6!}{2! \times 2!} = \frac{720}{4} = 180$ ways

Now 4 vowels can be arranged in the 7 spaces (blanks) as shown by in the figure.

That way, no two vowels will come together.

This can be done in $\frac{7P_4}{2! \times 2!} = \frac{7!}{3! \times 2! \times 2!} = \frac{7 \times 6 \times 4 \times 5 \times 3!}{3! \times 2! \times 2!}$ = 7 × 6 × 5

= 210 ways

Total ways = $180 \times 210 = 37800$.

In which no two vowels will come together.

Common Solution [41-45]:

Statement number 4 in the common data, actually tells us that Cathy does not work on Monday and Saturday. So, arranging the given data in the table, we get

Let us denote Alex \Rightarrow A | Bhabha \Rightarrow B | Cathy \Rightarrow C | Dilip \Rightarrow D | Ethan \Rightarrow E

	Mon	Tue	Wed	Thu	Fri	Sat	Sun	
Α	× 1	√ 1	√ 1	× 1	× 4	√ 5	× 4	
В	√ 6	× 1	× 1	√ 8	× 7	× 5	√ 8	
С	× 2	× 1	× 1	√ 3	√ 3	× 2	× 3	
D	× 1	√ 1	√ 1	× 1	× 7	× 5	√ 8	
Е	√ 6	× 1	× 1	× 7	√7	√ 5	× 7	

Also note, in this question, we need to consider Sun – Mon as pair of consecutive days because the showroom is open on all 7 days of the week.

Step 1: A and D works together on Tue & Wednesday. It means no other person will work on these 2 days.

Also, they both must not be working on Monday & Thursday.

Step 2: C does not work on either on Monday or Saturday.

Means she will not work on Monday and Saturday.

Step 3: C is left with only 3 days. Thus she will work on these days i.e. Thursday & Friday (consecutive) & (third working day)

Step 4: It is given that C does not work with A on any day. It means A will not be working on Friday and Sunday as C is working.

Step 5: A must be working on Saturday as third day. B and D will not be working with him (Given). Thus E will definitely work on Saturday.

Step 6: On Monday A, C & D are not working means B & E will definitely & work on Monday.

Step 7: As E is working on Monday and Saturday, he cannot work on Thursday, because in that case he will not be working on 2 consecutive days.

Also he cannot work on Sunday because in that case he will work on 3 consecutive days.

So the only option left for him is Friday along with Monday & Saturday.

Step 8: It is evident from the table that only B, C and D will be working on Sunday.

This way the table is completed and we can answer all the questions.

- 41. **(d)** Bhabha and Cathy work together for 2 days.
- 42. (d) The consecutive days Ethan works on Friday & Saturday.
- 43. (c) Dilip and Ethan do not work together at all.
- 44. **(b)** Bhabha, Cathy and Ethan work on Sunday.
- 45. **(a)** One of the days Alex work on is Saturday.
- 46. (c) option C. The difference between climacteric and non-climacteric fruits is referred to in the third line of the first paragraph. Since this difference matters to fruit growers and greengrocers who must make sure their wares are in tip-top condition when they arrive at the marketplace, it can be safely said that commercial aspect of the interest is being discussed here.
- 47. (d) option 4. First line of the second paragraph clearly states that fruits exist to solve a problem faced by all plants how to spread their progeny around. Hence, option 4.
- 48. (a) option A. Let's understand the demand of the question here. The third para clearly suggests that two researchers studied 80 varieties of fruits. The second last line of the third paragraph shows that 15 varieties of these fruits were eaten by ground dwellers and 21 varieties were eaten by arboreal animals. This was the first finding of that study. Now, the first line of the next paragraph suggests that their hypothesis was fortified by **other evidence** that is non climacteric fruits have vivid colours.



So finding II (Thirty-six varieties of climacteric and non-climacteric fruits were eaten predominantly either by grounddwellers, or by arboreal or aerial animals respectively) is clearly the main evidence and finding I (Non-climacteric fruits tend to have vivid colours) is the 'other' or secondary evidence. Hence the correct answer is option a.

- 49. (d) option (d). Refer to the first line of the last paragraph. It is clearly stated that they looked at the fruits eaten by the people (for the study).
- 50. (b) option (b). Refer to the last line of the first paragraph It clearly states that how those differences (between climacteric and nonclimacteric originally came about remains unclear.
- 51. (b) On the 14th of May 1796, vaccine matter was taken from the hand of a dairy maid, Sarah Nelmes, and inserted by two superficial incisions in the arms of James Phipps, a healthy boy of about eight years of age. The boy went through an attack of cowpox as expected. After this, however, it was necessary to determine whether he was protected from smallpox. After waiting two months Jenner inoculated him with material from a smallpox patient. He was delighted to note that the boy was not affected by smallpox.
- 52. (b) Local reaction means a reaction occurring at the point of stimulation of injection of foreign substances.
- 53. **(d)** Refer to the first line of the last paragraph. The paragraph clearly states that Jenner's discovery was much appreciated by his contemporaries because of which he was given many honours and diplomas.
- 54. (b) Refer to the first line of the second paragraph, "By 1798, Jenner had succeeded in demonstrating the protective quality of the cowpox virus against smallpox, by putting on record details of 23 cases, contracted either casually or by direct inoculation."
- 55. (b) Refer to the third and fourth line of the fourth paragraph, "Dr. Ingenhouz, became the leader of a strong faction of the medical profession of London, who not only would have nothing to do with vaccination, but proclaimed openly that it was a dangerous innovation, absolutely unjustifiable, and communicated a disease without protecting against any other."
- 56. (c) The entire passage talks about how Dr. Jenner went that extra mile to test his discovery and make it known.

Option (a) – the passage doesn't talk about likelihood of scientific breakthroughs happening in major urban centers. Therefore, this option can be eliminated.

Option (b) – The central idea of the passage is clearly the discovery of Dr. Jennings and his struggle to demonstrate it assertion to his contemporaries. The passage doesn't talk about importance of documentation for disseminating Science.

Option (d) – can be eliminated for the same reasons. The reluctance to accept the vaccine isn't the central idea of the passage either.

- 57. (a) Refer to the third line of the third paragraph, "He remained in London for nearly three months without being able to find anyone who would submit to vaccination. Jenner went back to Gloucestershire, disappointed."
- 58. (d) The phrase 'Piece of Cake' is used to describe something that is very easy to complete. The other options are not valid phrases to be used in this context.
- 59. (b) Option(b). A taste of one's own medicine means harsh or unpleasant treatment that is like the treatment someone has given other people.
- 60. (c) option (c). look on the bright side means to be optimistic or cheerful in spite of difficulties. Options a and b are wrong because the correct expression is 'Look on the bright side'. Option d is not a valid phrase to be used here.
- 61. (a) Mindfulness the quality or state of being conscious or aware of something Inclusivity - creating an environment where all people are truly welcomed, valued and respected Diversity is a group of people who are different in the same place Discretionary - used as desired or needed Discriminatory - unfair According to the given context the chain of coffee shops has been committed to building a culture where everyone is welcome.

So, the option that fits this best is inclusivity and diversity which means creating an environment where all people are truly welcomed, valued and respected.

- 62. (b) condescendingly showing or characterized by a superior attitude toward others Ambitiously – with the intention of meeting high aspirations Overly – excessively Controversially - in a way that causes disagreement or discussion Option a fits the context best.
- 63. (d) *Hidden in plain sight* means to be unnoticeable, by staying visible in a setting that masks presence; camouflage The given information is about spies, sending information to their heads, disguised as harmless looking secretaries or researchers. So they hide in plain sight which means blend in with their surroundings so well that it goes by unnoticed. The other options are irrelevant.
- 64. (c) looks down on regard someone with a feeling of superiority
 Looks up to admire and respect someone
 Looks towards To glance, gaze, or turn one's face in the direction of someone or something
 Looks away from To turn one's face away (from someone or something)
 Since she's rich, she probably regards others with a feeling of superiority. Hence option C.
 (1) Picket and the fact of the fact of
- 65. **(b)** Pick up take hold of and lift or move someone or something; raise or arouse
- 66. (c) This is a third conditional sentence. To make a sentence in the third conditional, we use, If + past perfect, would/wouldn't have + past participle. If you had told me about the meeting, I would have come. Hence only option c fits the given requirement. Besides, the other options are grammatically wrome.



67. (d) This is a case of inversion which means a reversal of position, order, form, or relationship: the placement of a verb before its subject for example, little did they know about me.

- 68. (a) Provided can emphasize a condition. Follows the same structure as 'if' conditional sentence. Provided + simple present, (followed by) simple present
- 69. (b) Revert return to former state or activity
 - Its shows possession
 - It's contracted form of it is
 - The word 'back' after the word 'reverted' is redundant.
- 70. **(b)** The central idea of the paragraph is shopping. Since the first line is missing, we need to find something that can work as a passage opener. Option B fits the context better as it talks about how culture plays an important role in the shopping experience and the next sentence talks about how climate and culture affect the shopping experience of people of SE Asia and Europeans.

Option (a) – is about promotional discounts to entice shoppers. Doesn't relate to the theme of the passage.

Option (b) – talks about role of culture in shaping the shopping experience.

Options (c) – is about how people enjoy shopping and comparing brands. But this sentence doesn't relate to the next sentence which shows the difference in the shopping habits of people of Southeast Asia and Europeans. So this one should be ruled out.

Option (d) – related to the last sentence of the passage more than the first sentence.

71. **(b)** The paragraph talks about Goa and how reaching there is not a major problem. The sentence that best fits the missing part that follows is option B (There are buses that ply at night, there are both day and night trains, and one can always hire a car and reach there in comfort in about 10 hours).

Option (a) though talks about a variety of things that one can do in Goa won't be a good fit here as the line preceding the missing part talks about reaching Goa and the line that follows is about its beaches.

Option (c) – It looks like a good choice as it talks about popular Goan beaches but the following line starts with "the major Goan beaches" instead of "these Goan beaches". This shows that the missing line is not connected to the Goan beaches. Option (d) – can be the last line of the given paragraph.

- 72. **(b)** options (a) and (d) can be ruled out because these do not relate to the next statement that discusses 'the causes for this situation'. Between options (b) and (c), option (b) is a better answer choice as it relates to the given information perfectly.
- 73. (a) The entire paragraph is about the script which the scholars have been unable to decipher. Keeping this in mind, option (b), (c) and (d) should be ruled out as these are about trade, nature of relationship and age of civilization. Only option (a) is about decoding the language.
- 74. **(b)** Universe (n)– cosmos

Ideologies - a set of ideas which form the basis for a political or economic system

Phenomenon (singular) - an observable fact or event ; Phenomena (plural)

Configuration (n) - the way in which the parts of something, or a group of things, are arranged

Option (d) can be ruled out as the word configuration would be a misfit in the first blank.

Option (a) can be ruled out as achieving spaceflights helps observe objects and 'ideologies' that are better observed from a space perspective, probably not; it won't be parallel.

Option (c) can be ruled out as the aforementioned activities aren't the elements of space 'identification'.

Let's place option B in the blanks and see its impact

Achieving spaceflight enabled humans to begin to explore the solar system and the rest of the universe , to understand the many objects and phenomena that are better observed from a space perspective, and to use for human benefit the resources and attributes of the space environment. All of these activities - discovery, scientific understanding, and the application of that understanding to serve human purposes – are elements of space exploration.

75. (b) Monochrome (adj)- using only black, white and shades of grey

Barely (adv)- only just; almost notMisconception (n) – wrong idea or understanding of something.

Accusation (n)- a charge of wrong doing.

Classic (adj)- typical

Dispassionately (adj) - in an unemotional, rational, and impartial manner

Option (d) can be ruled out as the movies can't be preserved 'dispassionately'.

Option (c) will be ruled out as accusation will be a misfit in the third blank. If the films aren't preserved well, it will probably lead to some conclusion not accusation.

Option (a) will be ruled out as barely won't fit as beautifully and meaningfully as poorly in the second blank.

76. **(d)** Educational (adj) – providing or relating to education

Firewall - A firewall is a network security device that monitors incoming and outgoing network traffic and decides whether to allow or block specific traffic based on a defined set of security rules hackers (n) - a person who uses a computer to look at and/or change information on another computer without permission Inescapable(adj) – that can't be avoided. Unsolicited – not asked for; uninvited

Regulators – a person or thing that regulates/controls something

Spammer (n) - a person or organization that sends irrelevant or unsolicited messages over the internet, typically to large numbers of users, for the purposes of advertising, phishing, spreading malware, etc.

Let's evaluate the options for the first blank. The messages are promotional (done to advertise your products) that means these won't be educational or informative so we can rule out options (a) and (c). unsolicited (not all ed for) looks like a

answer choice than inescapable (something that can't be avoided). Hence, option (d) is the perfect answer choice for this question.

- 77. (c) Treacherously (adv) – in a disloyal and faithless manner Enormously (adv)- to a great degree or extent Alleged (adj)- accused but not proven or convicted Insider (n) – a person who knows a lot about a group or an organization because he's a part of it. Illicitly (adv)- in a way that is contrary or forbidden by law, rules or custom Accused (n)– a person who has been arrested for or formally charged with a crime Dramatically (adv) - something done with great flare or done in an overly exaggerated or theatrical manner. Though grammatically all the options look alright, we need to carefully evaluate them. Now, instead of announcing, the investigator would probably allege or even accuse that someone has leaked the information. Hence, options (a) and (b) can be eliminated. If we look at the third blank, the associate made profits illicitly (Illegally) not dramatically. 78. (a) Orthography - the system of spelling in a language Linguistics - the scientific study of language **Cogitate** - think deeply about something; meditate or reflect. Ponder/Mull - to think about something carefully or for a long time Options (b), (c), and (d) can be ruled out because the words language, animals and writing won't fit in the second blank. One wouldn't want to think deeply about these and life in general. Option (a) looks like a good answer choice. The beauty of philosophy is that it poses thought-provoking questions and inspires us to think deeply about ourselves and life in general. Further, pondering about the ideas and perspectives outlined by different thinkers can help us to gain a better understanding of the world we live in. The sentences below have words that are missing. Choose the best option from those given below to complete the sentence. 79. (c) Belief - a firm thought that something is true, often based on revelation Worldview - a comprehensive conception or apprehension of the world especially from a specific standpoint The paragraph begins with an anthropocentric view where humankind is the central or most important element of existence. Since, this can't be a construct (something that is created, often with a complex form) or universe (cosmos), option a and d can be eliminated. So we're left with option b and c. Now, let's take a look at the third blank. The idea about killing animals painlessly can be 'explicitly' (clearly stated) and not 'partially' on some widely accepted idea. (c) Spot (n) - a small dirty mark on something 80 Spot - location Stain on career - something bad that someone has done that spoils their reputation Spot on (adj)- exactly right Sentences (a), (b), and (d) have used the words correctly. Stain on career would be a better fit instead of spot on his career in sentence (c). 81. (a) crass (adj) - stupid, showing that you do not understand something Cross - to go from one side of something to the other Bear the cross - a problem that causes trouble or worry for someone over a long period of time Cross - angry or annoyed Sentence (b), (c), and (d) have used the word cross appropriately. Crass should be used instead of cross in sentence a. 82. (d) flout - to refuse to obey or accept something Flaunt - to show something that you are proud of so that other people will admire it Sentence (a), (b) and (c) have used the word flout correctly. Flaunt should be used in sentence 4 to give the sentence its correct meaning. 83. (d) State (v) – to say or write something, especially formally State (n) - a condition or way of being State (n) - a division under the Indian constituency The word state is correctly used in the first three sentences. The demeanour (behaviour towards others) can't be of great 'state'. Hence, the word state has been inappropriately been used in option D. 84. Draw as used in sentence (a) means to pull something out of its resting place (c) Draw as used in sentence (b) means an act of deciding something by chance by pulling out names or numbers from a bag, etc. Draw as used in sentence (d) means a result of a game or competition in which both players and teams get the same score so that neither of them wins. Drawer - a container which forms part of a piece of furniture such as a desk that you can pull out to put things in. The word drawer should be used in place of draw in sentence (c) to give the sentence its correct meaning. 85. (d) Collaborate - to work together (with somebody), especially to create or produce something Corroborate - to support a statement, idea, etc. by providing new evidence Sentence (a), (b) and (c) have used the word collaborate correctly. The word, however, to be used in sentence (d) should be corroborate. (1432) Sentence 1 introduces the Central African Republic which has approved Bitcoin as an official currency. Sentence 4 builds on 86. it as 'this move' is criticized by the economists. Sentence 3 further talks about the opinion of the experts. Sentences 2 will follow sentence 3 as it states what the others claim about the overall impact of this move.
- 87. (31542)Sentence 3 clearly works as a passage opener as it introduces the city's Talao (Lake). Sentence 1 is a connected statement as it describes its location. Sentence 3 gives more information about the lake which was known natural water tank with the Sion



hillock expanding around it like an enclosing wall. Sentence 4 states that the lake has lost its prominence over the years and sentence 2 is about the present day state of the lake which is now surrounded by high-rise residential buildings.

- 88. **(43512)**Sentence 4 and 3 are paired sentence, sentence 4 being the passage opener as it introduces the June-to-September rain-bearing system which is the lifeblood of the country's economy. This is because the Indian population depends upon farm-derived income as stated in sentence 3. Sentence 5 and 1 are connected statements as these talk about 40% of India's net sown area which does not have access to irrigation and how a subpar (below a usual or normal level) monsoon cuts farm yields, output, and farm incomes thereby increasing the country's dependence on food imports. Sentence 2, however, gives a solution to the aforementioned.
- 89. **(4132)** Sentence 4 opens the passage as it talks about the ceramics, known as Yakimono, made by Japanese artisans. Sentence 1 is connected to sentence 4 as it talks about the popularity of 'these creations' i.e. the ceramics referred to in the first sentence. Sentence 3 and 2 are mandatory pairs as sentence 3 refers to the ceramics displayed in the touristy shops and sentence 4 talks about savvy tourists who find the best bargains in Tokyo's Kitchen Town.
- 90. (54321)Sentence 5 works as the passage opener when it talks about how human eye performs jiggles i.e moves quickly from side to side. Sentence 4 builds on it as it talks about how our eyes quickly scan the surroundings and sends the data to the brain. Sentence 2 and 3 are paired sentences as these talk about how youthful eyes jiggle regularly to take in new or unfamiliar stimuli and how the eye muscles grow slower and the pathways between the eye and the brain grow longer, more complex, or, in some cases, get damaged when the person attached to those eyes ages. This is connected to sentence 1 as it talks about its impact on the brain.

