# MATHEMATICS

**1.** Set A contains 5 elements $a_1, a_2, a_3, a_4, a_5$ with mean & variance 5 & 12 respectively. Set B contains 5 elements $b_1, b_2, b_3, b_4, b_5$ with mean 8 & variance 20. A set C is made of elements $a_i - 3, i \in \{1, 2, 3, 4, 5\}$ and $b_i + 2, i \in \{1, 2, 3, 4, 5\}$. Find sum of mean & variance of elements of set C.

(1) 38 (2) 40 (3) 42 (4) 36

**Ans.** **(1)**

**Sol.** $\sum a_i = 25$ $\sum b_i = 40$

$\frac{\Sigma a_i^2}{5} - (\bar{a})^2 = 12$ $\frac{\Sigma b_i^2}{5} - (\bar{b})^2 = 20$

$\sum a_i^2 = 185$ $\sum b_i^2 = 420$

$$\bar{c} = \frac{10+50}{10} = 6$$

$$\sigma^2 = \frac{\Sigma c_i^2}{10} - (6.5)^2 = \frac{\Sigma(a_i-3)^2 + \Sigma(b_i+2)^2}{10} - (6)^2$$

$$= \frac{\Sigma a_i^2 + \Sigma b_i^2 - 6\Sigma a_i + 4\Sigma b_i + 65}{10} - (6)^2$$

$$= \frac{185+420-150+160+65}{10} - (6)^2 = 32$$

$\therefore$ Mean + Variance = 38

**2.** How many truth value of statement $(p \vee q) \wedge (p \vee r) \rightarrow (q \vee r)$ is true

(1) 5 (2) 7 (3) 6 (4) 8

**Ans.** **(2)**

**Sol.**

| p | q | r | $(p \vee q) \wedge (p \vee r)$ | S |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | T | T |
| F | T | T | T | T |
| F | F | T | F | T |
| T | T | F | T | T |
| T | F | F | T | F |
| F | T | F | F | T |
| F | F | F | F | T |

**3.** Let $f(x) = [x^2 - x] + |-x + [x]|$ is

(1) continuous at $x = 0$ (2) continuous at $x = 1$

(3) differentiable at $x = 0$ (4) None of these

**Ans.** **(2)**

**Sol.** $f(0) = 0$

$f(0^+) = \lim_{n\to 0}[h^2 - h] + |-h + [h]|$

$= -1 + 0$

$= -1 \neq f(0)$

$\therefore$ discontinuous at $x = 0$

$f(1) = 0 + 0$

$f(1^+) = \lim_{n\to 0}[(1+h)^2 - (1+h)] + |-(1+h) + [1+h]|$

$= \lim_{n\to 0}[h + h^2] + |-1 - h + 1|$

$= 0 + 0 = 0$

$f(1^-) = \lim_{n\to 0}[(1-h)^2 - (1-h)] + |-(1-h) + [1-h]|$

$= \lim_{n\to 0}[-h + h^2] + |h - 1|$

$= -1 + (1) = 0$

**4.** The number of integral values of x satisfying the inequation $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is

(1) 5 (2) 6 (3) 7 (4) 8

**Ans.** **(2)**

**Sol.** Case-I : $x + \frac{7}{2} > 1 \Rightarrow x > -\frac{5}{2}$ .....(i)

$\left(\frac{x-7}{2x-3}\right)^2 \geq 1 \Rightarrow \left(\frac{x-7}{2x-3}+1\right)\left(\frac{x-7}{2x-3}-1\right) \geq 0$

$\frac{(3x-10)}{(2x-3)^2}(-x-4) \geq 0$

$\frac{(3x-10)(x+4)}{(2x-3)^2} \leq 0$

$x \in \left[-4, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$

After intersection with (i) $\Rightarrow x \in \left(-\frac{5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$

Case-II : $0 < x + \frac{7}{2} < 1 \Rightarrow -\frac{7}{2} < x < -\frac{5}{2}$ .....(ii)

$0 < \left(\frac{x-7}{2x-3}\right)^2 \leq 1 \Rightarrow x \in (-\infty, -4] \cup \left[\frac{10}{3}, \infty\right) - \{7\}$

After intersection with (ii) $\Rightarrow x \in \phi$

$\therefore$ (Case-I) $\cup$ (Case-II) $\Rightarrow x \in \left(-\frac{5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$

$\therefore$ Number of integral values = 6

**5.** Let $a_1, a_2 \ldots\ldots a_{100}$ be anAP whose first term is 2 and mean is 200. If $b_i = i(a_i - i)$, then mean of $b_1, b_2 \ldots\ldots b_{100}$ is

(1) 10050 (2)10049.5

(3)20099 (4) 20049.5

**Ans. (2)**

**Sol.** $200 = \frac{a_1 + a_2 + \ldots\ldots\ldots\ldots + a_{100}}{100}$

$200 = \frac{100}{2} \times \frac{1}{100}\left(2\times2+(100-1)d\right)$

$396 = 99d \quad \Rightarrow d = \frac{396}{99} = 4$

Required mean = $\frac{1}{100}\sum_{i=1}^{100} i\left(a_i - i\right)$

$= \frac{1}{100}\sum_{i=1}^{100} i\left(2+(i-1)d - i\right)$

$= \frac{1}{100}\left\{(2-d)\frac{100\times101}{2} + (d-1)\frac{100\times101\times201}{6}\right\}$

$= \frac{101}{2}\left\{2-d+(d-1)67\right\}$

$\frac{101}{2}\left\{66\times\frac{396}{99} - 65\right\} = \frac{101}{2}\{199\} = 10049.5$

**6.** If a and b are roots of quadratic equation $x^2 - 7x - 1 = 0$ then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to

(1) 51 (2) 34 (3) 68 (4) 45

**Ans. (1)**

**Sol.** $\dfrac{a^{19}\left(a^2+\dfrac{1}{a^2}\right)+b^{19}\left(b^2+\dfrac{1}{b^2}\right)}{a^{19}+b^{19}}$

$\because a^2-7a-1=0$

$a^2-1=7a$

$a-\dfrac{1}{a}=7$

$a^2+\dfrac{1}{a^2}=51$

$\dfrac{51a^{19}+51b^{19}}{a^{19}+b^{19}}=51$

**7.** Number of solution of equation $3\cos^4\theta-4\cos^2\theta-\sin^6\theta+1=0$, $\theta\in[0,2\pi]$ is

(1) 5 (2) 6 (3) 7 (4) 8

**Ans. (1)**

**Sol.** put $\cos^2\theta = t$

$3t^2-4t-(1-t)^3+1=0$

$\Rightarrow \quad 3t^2-4t+t^3-1-3t(t-1)+1=0$

$\Rightarrow \quad t^3-t=0$

**Case-I** $\cos^2\theta=0$

$7\ \theta\in\left\{\dfrac{\pi}{2},\dfrac{3\pi}{2}\right\}$

**Case-II** $\cos^2\theta=0$

q Î {0, π, 2π}

**Case-III** $\cos^2\theta=-1$

Rejected

$\therefore$ No of solution = 5

**8.** 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seat. Find number of such seating arrangement.

(1) 18 (2) 9 (3) 45 (4) 44

**Ans. (4)**

**Sol.** $D(5)=5!\left(\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}-\dfrac{1}{5!}\right)$

$=60-20+5-1$

$=44$

**9.** If x + y + 3 = 17, x, y, z $\in$ whole number Find number of solutions.

(1) 145 (2) 154 (3) 171 (4) 181

**Ans.** **(3)**

**Sol.** Number of solution = $^{(17+3-1)}C_{(3-1)}$

$= {}^{19}C_2 = 19 \times 9 = 171$

**10.** Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ..... + \frac{1}{5^{108}}$ , then find the value of $(16S - 25^{-54})$, then S is equal to

**Ans.** **(2175)**

**Sol.** $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ..... + \frac{1}{5^{108}}$

$\frac{5}{5} = \frac{109}{5} + \frac{108}{5^2} + .... + \frac{1}{5^{109}}$

$\frac{45}{5} = 109 - \left(\frac{1}{5} + \frac{1}{5^2} + ..... + \frac{1}{5^{109}}\right)$

$\frac{45}{5} = 109 - \frac{1}{5}\left(\frac{1 - \left(\frac{1}{5}\right)^{109}}{\left(1 - \frac{1}{5}\right)}\right)$

$\frac{45}{5} = 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$

$S = \frac{545}{4} - \frac{5}{16}\left(1 - \frac{1}{5^{109}}\right)$

**11.** Let the image of point (8, 2, – 4) about the plane 2x – y – 3z = 12 is ($\alpha, \beta, \gamma$). Then $\alpha + \beta + \gamma$ is

(1) 8 (2) 10 (3) 12 (4) 14

**Ans.** **(2)**

**Sol.** $\frac{x-8}{2} = \frac{y-2}{-1} = \frac{z+4}{-3} = -2\left(\frac{16-2+12-12}{14}\right)$

$\Rightarrow \alpha = 4, \beta = 4$ and $\gamma = 2$

**12.** In the expansion of $(x + 2)^9$ the mean of coefficient of x, $x^2$, $x^3$, ....... $x^9$ is

(1) $\frac{3^8}{10}$ (2) $\frac{3^6}{10}$ (3) $\frac{3^7}{5}$ (4) $\frac{3^9}{10}$

**Ans.** **(4)**

**Sol.** $T_{r+1} = {}^9C_r\, 2^{9-r}.\, x^r$

$\text{mean} = \frac{1}{10}\sum_{r=0}^{9} {}^9C_r 2^{9-r}$

$= \frac{1}{10} \times 2^9 \left(1 + \frac{1}{2}\right)^9 = \frac{3^9}{10}$

**13.** Let $M = [a_{ij}]_{2\times2}$, $0 \le i, j \le 2$ where $a_{ij} \in \{0, 1, 2\}$ and A be the event such that M is invertible then P(A) is

(1) $\frac{49}{81}$ (2) $\frac{50}{81}$ (3) $\frac{47}{81}$ (4) $\frac{46}{81}$

**Ans.** **(2)**

**Sol.** $|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \ne 0$

If $ad = 0 \Rightarrow bc = 0 \quad \Rightarrow \quad 25$

$ad = 1 \Rightarrow bc = 1 \Rightarrow 1$

$ad = 2 \Rightarrow bc = 2 \Rightarrow 4$

$ad = 4 \Rightarrow bc = 4 \Rightarrow 1$

$\therefore$ required probability $= 1 - \frac{31}{81} = \frac{50}{81}$

**14.** Integrate $\int_{-\ln 2}^{\ln 2} e^x \left(\ln(e^x + \sqrt{1+e^{2x}})\right) dx$ equal to

(1) $\ln\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{1/3}} + \frac{3\sqrt{5}}{4}$

(2) $\ln\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{1/2}} - \frac{\sqrt{5}}{4}$

(3) $\ln\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{1/2}} + \frac{\sqrt{5}}{2}$

(4) $\ln\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{1/2}} - \frac{\sqrt{5}}{2}$

**Ans.** **(4)**

**Sol.** Let $e^x = t$

$$\int_{1/2}^{2} \ln(t+\sqrt{1+t^2})\, dt$$

$$= t\ln(t+\sqrt{1+t^2})\Big|_{1/2}^{2} - \int_{1/2}^{2} \frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$$

$$= 2\ln(2+\sqrt{5}) - \frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right) - \int_{1/2}^{2} \frac{t}{\sqrt{1+t^2}}\, dt$$

$$= 2\ln(2+\sqrt{5}) - \frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right) - \int_{\sqrt{5}/2}^{\sqrt{5}} dt$$

$$= \ln\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{1/2}} - \frac{\sqrt{5}}{2}$$

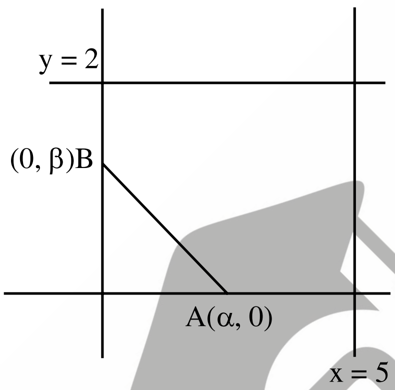
**15.** A rectangle is formed by the line $(x - 5)(y - 2) = 0$ along with co-ordinate axes. Let two points A & B move along co-ordinate axes starting at origin. At any time position of A & B are $(\alpha, 0)$ & $(0, \beta)$, $0 < \alpha < 5$, $0 < \beta < 2$. The locus of mid-point of line segment AB such that it divides area of rectangle in 4 : 1 ration

(1) straight line
(2) ellipse
(3) parabola
(4) hyperbola

**Ans.** **(4)**

**Sol.** $h = \frac{\alpha}{2}$ & $k = \frac{\beta}{2}$

$$4 = \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta}$$




$\frac{5}{2}\alpha\beta = 10$

$(2h)(2k) = 4$

$\therefore$ locus is $xy = 1$

**16.** Let $f'(x) . (x \log x) + f(x) \log x + f(x) \geq 1$ such that $f(2) = \frac{1}{2}$, $f(4) = \frac{1}{4}$ and

S1 : $f(x) \leq 1 \ \forall x \in [2, 4]$

S2 : $f(x) \leq \frac{1}{8} \ \forall x \in [2, 4]$

(1) Only S1 is true
(2) Only S2 is true
(3) Both $S_1$ & $S_2$ are True
(4) Both S1 and S2 are false

**Ans.** (3)

**Sol.** $f'(x)\, x\, \ell n\, x + f(x)\, \ell n\, x + f(x) \geq 1$

$\frac{d}{dx}(x\, \ell n\, x\, f(x)) \geq 1$

$g(x) = x\, \ell n\, x\, f(x)$

$g(2) \leq g(x) \leq g(4) \Rightarrow 2(\ell n\, 2)\frac{1}{2} \leq x\, \ell n\, x\, f(x) \leq 4(\ell n\, 4)\frac{1}{4}$

$\ell n\, 2 \leq x\, \ell n\, x\, f(x) \leq \ell n\, 4$

$\frac{\ell n 2}{x \ell n x} \leq f(x) \leq \frac{\ell n 4}{x \ell n x}$

$x\, \ell n\, x \in [2\ell n\, 2, 4\, \ell n\, 4]$

$\Rightarrow \quad f(x) \leq \frac{\ell n 4}{2\ell n 2} = 1$

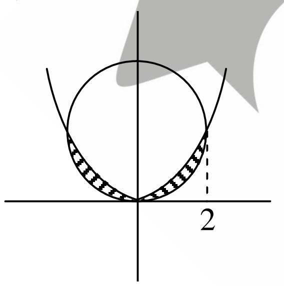
$f(x) \geq \frac{\ell n 2}{4 \ell n 4} = \frac{1}{8}$

**17.** Find area enclosed by $x^2 + (y-2)^2 \leq 4$ and $2y \leq x^2$

(1) $\left(2\pi - \frac{16}{3}\right)$
(2) $\left(2\pi + \frac{16}{3}\right)$
(3) $\left(\pi + \frac{16}{3}\right)$
(4) $\left(4\pi - \frac{16}{3}\right)$

**Ans.** **(1)**

**Sol.**




$$\text{Area} = 2\int_0^2 \left[\frac{x^2}{2} - \left(-\sqrt{4-x^2} + 2\right)\right] dx$$

$$= 2\left[\frac{8}{6} - 4 + \frac{\pi(4)}{4}\right]$$

$$= \frac{8}{3} - 8 + 2\pi$$

$$= \left(2\pi - \frac{16}{3}\right)$$

**18.** If $\frac{x^2}{1+n}+\frac{y^2}{3+n}=1$, then minimum value of n, where $n \in N$ such that eccentricity of conic is a rational number is

(1) 5 (2) 2 (3) 3 (4) 4

**Ans.** **(1)**

**Sol.** $e^2 = 1- \frac{1+n}{3+n}=\frac{2}{n+3}$

$\Rightarrow n \in \{5 .........\}$

**19.** The number of rational terms in $(3^{1/2} + 5^{1/4})^{680}$ is

(1) 171 (2) 181 (3) 191 (4) 151

**Ans.** **(1)**

**Sol.** $T_{r+1} = {}^{680}C_r(3^{1/2})^{680-r}(5^{1/4})^r$

$= {}^{680}C_r\ 3^{340-\frac{r}{2}}\ 5^{r/4}$

$\therefore r \in \{0, 4, 8 ....... 680\}$

**20.** Let A be a square matrix of order 2 such that $|A^2 - A| = 4$ and $A = \alpha A + I$. then sum of all possible real values of $\alpha$ is

**Ans.** **(1.50)**

**Sol.** $A' = \alpha A + I$

$\Rightarrow A = \alpha A' + I$

$\Rightarrow A = \alpha(\alpha A + I) + I$

$(1 - \alpha^2)\ A = (1 + \alpha)I$

Case $\alpha = -1$:

$A' + A = I$

$|A||-A'| = 4$

$|A| = \pm 2$

Case $\alpha \neq -1$:

$A = \frac{I}{1-\alpha}$

$|A||\alpha A'| = 4$

$\alpha\left|\frac{I}{1-\alpha}\right| = \pm 2$

$\frac{\alpha}{(1-\alpha)^2} = \pm 2$

$\therefore$ required sum $= 2 + \frac{1}{2} - 1$

$= \frac{3}{2}$

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