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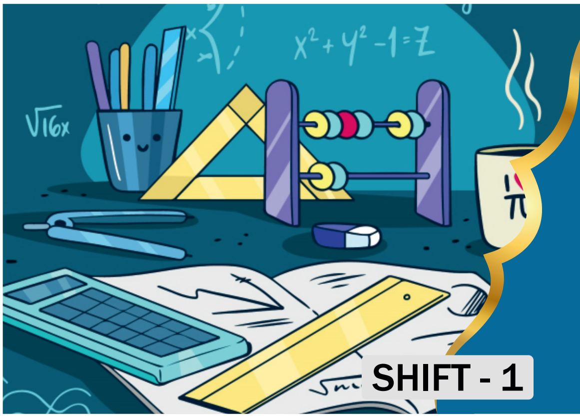
APRIL ATTEMPT

PAPER-1 (B.Tech / B.E.)



QUESTIONS & SOLUTIONS

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12 APRIL, 2023

9:00 AM to 12:00 Noon

Duration : 3 Hours | Maximum Marks : 300

SUBJECT - MATHEMATICS

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MATHEMATICS

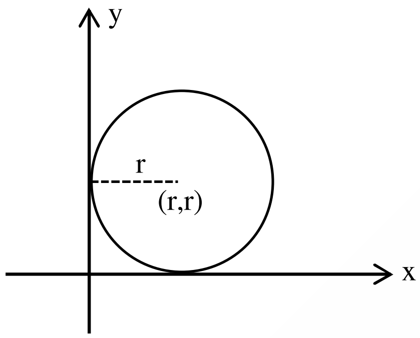
1. Two circles having radius r_1 and r_2 touch both the coordinate axes in first quadrant line x + y = 2 makes intercept 2 on both circle. The value of $r_1^2 + r_2^2 - r_1r_2$ is

(1) 8 (2) 7 (3) 5 (4) 2

Ans. **(2)**

Sol. $(x - r)^2 + (y - r)^2 = r^2$

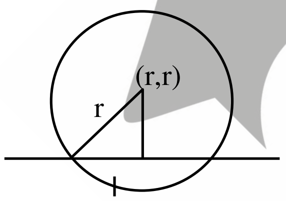
Distance of (r, r) from x + y = 2 is $\left|\frac{r + r - 2}{\sqrt{2}}\right| = \sqrt{2}\ |r - 1|$



$\left(\sqrt{2}\,|r-1|\right)^2 + 1 = r^2$

$\Rightarrow 2(r^2 - 2r + 1) + 1 = r^2$

$r^2 - 4r + 3 = 0 \Rightarrow r = 1, 3$



$r_1^2 + r_2^2 - r_1r_2 = 7$

2. If $\frac{^nC_n}{n+1} + \frac{^nC_{n-1}}{n} + ----- \frac{^nC_1}{2} + {^nC_0} = \frac{1023}{10}$ then the value of n is

(1) 9 (2) 7 (3) 5 (4) 4

Ans. **(1)**

Sol. $\sum_{r=0}^{n} \frac{^nC_r}{(r+1)} = \frac{1}{(n+1)} \sum_{r=0}^{n} {^{n+1}C_{r+1}}$

$\frac{1}{n+1}\left[2^{n+1} - 1\right] = \frac{1023}{10}$

$\Rightarrow \frac{2^{n+1} - 1}{n+1} = \frac{1023}{10} \Rightarrow n = 9$

3. The sum of coefficients of first 50 terms of the expression $(1 - x)^{100}$ is

(1) 0 (2) $^{99}C_{49}$ (3) $\frac{1}{2}\,^{100}C_{50}$ (4) $-^{99}C_{49}$

Ans. **(4)**

Sol. $(1-x)^{100} = {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2\, x^2 \ldots\ldots + {}^{100}C_{100}\, x^{100}$

$\because\ {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - {}^{100}C_3 + \ldots\ldots\ldots + {}^{100}C_{100} = 0$

$2\left({}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - {}^{100}C_3 + \ldots\ldots\ldots {}^{100}C_{49}\right) + {}^{100}C_{50} = 0$

${}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - {}^{100}C_3 + \ldots\ldots\ldots = -\frac{1}{2}\,{}^{100}C_{50}$

4. Number of 5-digit number formed using digits 0, 1, 3, 5, 7, 9 which are greater than 40,000 (repetition is not allowed) and divisible by 5 is equal to

(1) 100 (2) 120 (3) 240 (4) 360

Ans. **(2)**

Sol.

				5
2	4	3	2	1

\+

				0
3	4	3	2	1

Number of 5-digit number = $5 \times 6 \times 6 \times 6 \times 2 = 2160$

5. $\int_{-0.15}^{0.15} |100x^2 - 1|\, dx$ is equal to

(1) $\frac{23}{600}$ (2) $\frac{23}{300}$ (3) $\frac{23}{100}$ (4) $\frac{23}{120}$

Ans. **(4)**

Sol. $I = 2\int_{-0.15}^{0.15} |100x^2 - 1|\, dx$

$100x^2 - 1 = 0 \Rightarrow x = 0.1$

$I = 2\left[\int_0^{0.1} (1 - 100x^2)\, dx + \int_{0.1}^{0.15} (100x^2 - 1)\, dx\right]$

$I = 2\left[0.1 - \frac{100}{3}(0.1)^3 + \frac{100}{3}\{(0.15)^3 - (0.1)^3\}\right] - (0.05)$

$I = 2\left[0.1 - \frac{0.1}{3} + \frac{0.2375}{3} - 0.05\right] = 2\left[\frac{0.2875}{3}\right] = 0.19166 = \frac{23}{120}$

6. If $\Delta_k = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix}$ and $\sum_{k=1}^{n} \Delta_k = 216$, then 'n' is equal to

Ans. 6

Sol. $\sum_{k=1}^{n}\Delta_k = \begin{vmatrix} \sum_{k=1}^{n} 1 & \sum_{k=1}^{n}(2k-1) & \sum_{k=1}^{n} 2k \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix} = 216$

$= \begin{vmatrix} n & n^2+n-n & n^2+n \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix} = 216$

$= \begin{vmatrix} n & n^2 & n^2+1 \\ 0 & n+1 & -n \\ 0 & n & 0 \end{vmatrix} = 216$

$n^3 = 216 \Rightarrow n = 6$

7. Given $\vec{A} = \lambda\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $(\vec{A} + \vec{B} + \vec{C}) \times \vec{C} = 0$, $\vec{A}.\vec{C} = -9$, $\vec{B}.\vec{C} = -20$.

Find $|\vec{C} \times \lambda(\hat{i} + \hat{j} + \hat{k})|^2$ where $\lambda > 0$

(1) 62 (2) 61 (3) 60 (4) 33

Ans. **(1)**

Sol. $(\vec{A} + \vec{B}) \times \vec{C} = 0$

$\vec{C} = K(\vec{A} + \vec{B}) = K((\lambda + 2)\hat{i} - 2\hat{j} + 4\hat{k})$

$\vec{A}.\vec{C} = K(\lambda^2 + 2\lambda + 6) = -9$

$\vec{B}.\vec{C} = K(2\lambda + 4 + 2 + 12) = K(2\lambda + 18) = -20$

$\frac{\lambda^2 + 2\lambda + 6}{2\lambda + 18} = \frac{9}{20} \Rightarrow 10\lambda^2 + 11\lambda - 21 = 0$

$\lambda = 1$ as $\lambda > 0$, $K = -1$

Thus $\vec{C} = -(3\hat{i} - 2\hat{j} + 4\hat{k}) = -3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{C} \times \lambda(\hat{i} + \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -4 \\ 1 & 1 & 1 \end{vmatrix} = 6\hat{i} - \hat{j} - 5\hat{k}$

$|\vec{C} \times \lambda(\hat{i} + \hat{j} + \hat{k})|^2 = 36 + 1 + 25 = 62$

8. Let $S_1 : (p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction

$S_2 : (p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is tautology

(1) Both S_1 and S_2 are TRUE

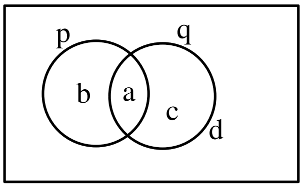
(2) Both S_1 and S_2 are FALSE

(3) S_1 is TRUE and S_2 is FALSE

(4) S_1 is FALSE and S_2 is TRUE

Ans. (1)

Sol. $S_1 \equiv (p \to q) \wedge (p \wedge \sim q) \equiv (a, c, d) \wedge (b) = \phi$ (Contradiction)



$S_2 \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv (a) \vee (c) \vee (b) \vee (d) \equiv (a, b, c, d)$

$\therefore$ tautology

9. Number of points of discontinuity of $\left|[x]\right| + \sqrt{x - [x]}$ on the interval $(-2,1)$ ([.] represents greatest integer function)

(1) 2 (2) 3

(3) 0 (4) 1

Ans. **(1)**

Sol. Let $f(x) = \left|[x]\right| + \sqrt{\{x\}}$

At $x = -1$, $f(-1^+) = |-1| + 0 = 1$

$f(-1^-) = |-2| + 1 = 3$

At $x = 0$, $f(0^+) = 0 + 0 = 0$

$f(0^-) = |-1| + 1 = 2$

Two points of discontinuity

10. If $(1 + x^2)dy = y(x - y)dx$ and $y(0) = 1$ then $y(2\sqrt{2})$ is

(1) $\dfrac{1}{\ln(\sqrt{2}+1)}$ (2) $\dfrac{3}{1+\ln(\sqrt{2}+1)}$

(3) $\dfrac{3}{1+2\ln(\sqrt{2}+1)}$ (4) $\dfrac{1}{1+\ln(3+2\sqrt{2})}$

Ans. **(2)**

Sol. $\frac{dy}{dx}=\frac{(yx-y^2)}{(1+x^2)}$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}=\frac{1}{(1+x^2)}\left(\frac{x}{y}-1\right)$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{x}{y}\frac{1}{(1+x^2)}=-\frac{1}{(1+x^2)}$

put $\frac{-1}{y}=t$

$\therefore \frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx}+\frac{tx}{(1+x^2)}=-\frac{1}{(1+x^2)}$

I.F. $e^{\int\frac{x}{(1+x^2)}dx} = e^{\frac{1}{2}\ln(1+x^2)} = \sqrt{1+x^2}$

Solution of D.E.

$t\sqrt{1+x^2} = -\int\frac{\sqrt{1+x^2}}{(1+x^2)}dx$

$=-\int\frac{dx}{\sqrt{1+x^2}}$

$-\frac{1}{y}\sqrt{1+x^2} = -\ln|x+\sqrt{x^2+1}|+C$

$y(0)=1 \Rightarrow -1=-\ln(1)+C \Rightarrow C=-1$

$-\frac{\sqrt{1+x^2}}{y}=-\ln|x+\sqrt{1+x^2}|-1$

at $x=2\sqrt{2}$, $-\frac{3}{y}=-\ln|2\sqrt{2}+3|-1$

$\Rightarrow y=\frac{3}{1+\ln(3+2\sqrt{2})}=\frac{3}{1+\ln(\sqrt{2}+1)^2}$

$y=\frac{3}{1+2\ln(\sqrt{2}+1)}$

11. Let $\vec{x}=a\hat{i}+\hat{j}+\hat{k}$, $\vec{y}=\hat{i}+b\hat{j}+\hat{k}$, $\vec{z}=\hat{i}+\hat{j}+c\hat{k}$. If $\vec{x}$, $\vec{y}$ and $\vec{z}$ are coplanar then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is $(a, b, c \neq 1)$

(1) 1 (2) –1 (3) 2 (4) –2

Ans. **(1)**

Sol. $[\vec{x}\ \vec{y}\ \vec{z}] = 0 \Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1 \begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$

$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(-(b-1)) = 0$

divide by $(1-a)(1-b)(1-c)$

$\frac{a}{(1-a)} + \frac{1}{(1-b)} + \frac{1}{(1-c)} = 0$

$\Rightarrow \frac{a}{1-a} + 1 + \frac{1}{1-b} + \frac{1}{1-c} = 1$

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

12. Let $S_n = \frac{n^2+3n}{(n+1)(n+2)}$ and $28 \sum_{k=1}^{10} \left(\frac{1}{a_k}\right) = P_1 . P_2 . P_3 .... P_m$ ($P_1, P_2, ... P_m$ are prime numbers), then value of m is (Where a_n general terms of series and S_n is sum of n terms of series)

(1) 5 (2) 6 (3) 7 (4) 9

Ans. **(2)**

Sol. $a_n = S_n - S_{n-1} = \frac{n^2+3n}{(n+1)(n+2)} - \frac{(n-1)^2+3(n-1)}{n(n+1)}$

$a_n = \frac{n^3+3n^2-(n+2)[n^2+n-2]}{n(n+1)(n+2)} = \frac{4}{n(n+1)(n+2)}$

$28 \sum_{k=1}^{10} \frac{k(k+1)(k+2)}{4} = 7 \sum_{k=1}^{10} k(k+1)(k+2)$

$S = \sum_{k=1}^{10} k(k+1)(k+2) = \sum_{k=1}^{10} \frac{k(k+1)(k+2)[(k+3)-(k-1)]}{4} = \frac{1}{4} . 10 \times 11 \times 12 \times 13$

So $\frac{7}{4} . 10 . 11. 12 . 13 = 2 . 3 . 5 . 7 . 11 . 13$

$m = 6$

13. Let the line x sin θ + y cos θ = 7 where $\theta \in \left(0, \frac{\pi}{2}\right)$. Point (α, 4) lies on locus of mid-point of intercepts made by line on coordinate axes then value of α is

(1) $\frac{28}{\sqrt{15}}$ (2) $\sqrt{15}$ (3) $\frac{23}{\sqrt{15}}$ (4) $\frac{14}{\sqrt{15}}$

Ans. **(1)**

Sol. x sin θ + y cos θ = 7

x-intercept is $\frac{7}{\sin\theta} \Rightarrow A\left(\frac{7}{\sin\theta}, 0\right)$

y-intercept is $\frac{7}{\cos\theta} \Rightarrow B\left(0, \frac{7}{\cos\theta}\right)$

locus of midpoint (h, k) is

$$h = \frac{7}{2\sin\theta}, k = \frac{7}{2\cos\theta}$$

as $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \frac{49}{4h^2} + \frac{49}{4k^2} = 1 \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{49}$

(α, 2) lie on it

$\frac{1}{\alpha^2} + \frac{1}{16} = \frac{4}{49}$

$\frac{1}{\alpha^2} = \frac{4}{49} - \frac{14}{16}$

$\frac{1}{\alpha^2} = \frac{64-49}{16\times 49} = \frac{15}{16\times 49} \Rightarrow \alpha = \frac{28}{\sqrt{15}}$

14. Let $z_0 = \frac{1}{2} + \frac{3i}{2}$ be the center of circle S of radius 1unit, $z_1 = 1 + i$, z_2 lies outside the circle S and z_0, z_1, z_2 are collinear. If $|z_1 - z_0|\,|z_2 - z_0| = 1$ then the smallest value of $|z_2|^2$ is

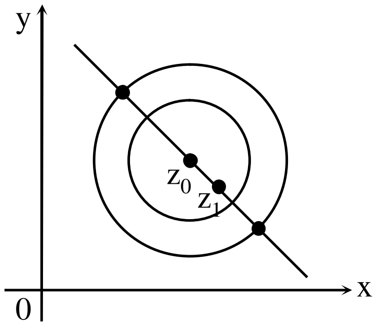
(1) $\sqrt{\frac{13}{2}}$ (2) $\sqrt{\frac{3}{2}}$

(3) $\sqrt{\frac{5}{2}}$ (4) 1

Ans. **(3)**

Sol. $|z_1 - z_0| = \left|\frac{1}{2} - \frac{i}{2}\right| = \frac{1}{\sqrt{2}} \Rightarrow |z_2 - z_0| = \sqrt{2}$

Now, z_0, z_1, z_2 ae collinear



Equation of line through z_0 and z_1 is

$z_0\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$ $\qquad y - 1 = -(x - 1) \Rightarrow y + x = 2$

$|z_2 - z_0| = \sqrt{2} \Rightarrow \left|x + iy - \frac{1}{2} - \frac{3i}{2}\right| = \sqrt{2}$

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 2$

$\left(x - \frac{1}{2}\right)^2 + \left(x - \frac{1}{2}\right)^2 = 2$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 = 1 \Rightarrow x = -\frac{1}{2}, \frac{3}{2}$

$x = \frac{3}{2} \Rightarrow y = \frac{1}{2}$, $x = -\frac{1}{2} \Rightarrow y = \frac{5}{2}$

z_2 can be $\left(\frac{3}{2}, \frac{1}{2}\right)$ or $\left(-\frac{1}{2}, \frac{5}{2}\right)$

$|z_2| = \sqrt{\frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{5}{2}}$ or $|z_2| = \sqrt{\frac{1}{4} + \frac{25}{4}} = \sqrt{\frac{13}{2}}$

15. Area enclosed by $y = x^3$ and tangent at $(-1,-1)$ to given curve is

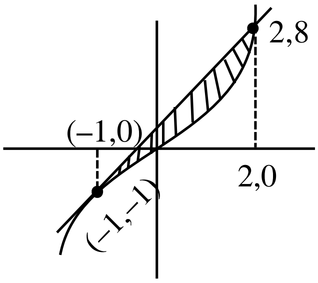
(1) $\frac{19}{4}$ (2) $\frac{21}{4}$

(3) $\frac{27}{4}$ (4) $\frac{25}{4}$

Ans. **(3)**

Sol. equation of tangent at (–1,–1)

$\frac{dy}{dx} = 3x^2$ slope at (–1,–1) is 3



tangent $y + 1 = 3(x + 1)$

$y = 3x + 2$

intersection $x^3 = 3x + 2 \Rightarrow x^3 - 3x - 2 = 0 \Rightarrow (x + 1)(x^2 - x - 2) = 0 \Rightarrow x = -1, (x + 1)(x - 2) = 0$

Area $= \int_{-1}^{2} \left(3x + 2 - x^3\right) dx = \left(\frac{3x^2}{2} + 2x - \frac{x^4}{4}\right)_{-1}^{2} = (6 + 4 - 4) - \left(\frac{3}{2} - 2 - \frac{1}{4}\right) = 6 - \left(\frac{6 - 8 - 1}{4}\right)$

$= 6 + \frac{3}{4} = \frac{27}{4}$

16. If roots of equation $x^2 + \sqrt{6}\,x + 3 = 0$ are α and β, then the value of $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is

(1) 3 (2) 9 (3) 81 (4) 729

Ans. **(3)**

Sol. $x^2 + 3 = -\sqrt{6}$

$x^4 + 6x^2 + 9 = 6x^2 \Rightarrow x^4 + 9 = 0$

$$\frac{\alpha^{14}(\alpha^9 + 1) + \beta^{14}(\beta^9 + 1)}{\alpha^{10}(\alpha^5 + 1) + \beta^{10}(\beta^5 + 1)} = \frac{(-729)\alpha^2(81\alpha + 1) + (-729\beta^2)(81\beta + 1)}{81\alpha^2(-9\alpha + 1) + 81\beta^2(-9\beta + 1)}$$

$$= -9\left[\frac{81\alpha^3 + \alpha^2 + 81\beta^3 + \beta^2}{-9\alpha^3 + \alpha^2 - 9\beta^3 + \beta^2}\right]$$

$\because\ \alpha + \beta = -\sqrt{6},\ \alpha\beta = 3 \Rightarrow \alpha^2 + \beta^2 = 0$

$$= -9\left[\frac{81(\alpha^3 + \beta^3)}{-9(\alpha^3 + \beta^3)}\right] = 81$$

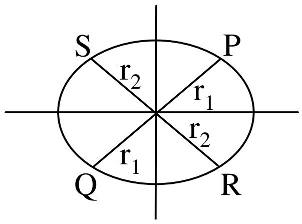
17. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$ be a point on the ellipse $9x^2 + 4y^2 = 36$. Three points Q, R, S lie on this ellipse.

If lines PQ and RS both pass through origin and PQ ⊥ RS then the value of $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2}$ is

(1) $\frac{13}{144}$ (2) $\frac{11}{144}$ (3) $\frac{26}{145}$ (4) $\frac{13}{141}$

Ans. **(1)**

Sol.



$m_{PQ} = \sqrt{3}$

$m_{RS} = -\frac{1}{\sqrt{3}}$

$PQ = 20P = 2\sqrt{\frac{4\times 3}{7} + \frac{36}{7}} = 2\sqrt{\frac{48}{7}}$

$(PQ)^2 = 4\left(\frac{48}{7}\right) = \frac{192}{7}$

Let $R(r_2\cos\theta, r_2\sin\theta) \equiv R\left(r_2\cos\left(-\frac{\pi}{6}\right), r_2\sin\left(-\frac{\pi}{6}\right)\right)$

$\equiv R\left(\frac{\sqrt{3}}{2}r_2, -\frac{1}{2}r_2\right)$

R lie on ellipse

$9\left(\frac{3}{4}r_2^2\right) + 4\left(\frac{1}{4}r_2^2\right) = 36$

$31\, r_2^2 = 144 \Rightarrow r_2^2 = \frac{144}{31}$

$(RS)^2 = (2r_2)^2 = \frac{4(144)}{31}$

$\frac{1}{PQ^2} + \frac{1}{RS^2} = \frac{31}{4(144)} + \frac{7}{192} = \frac{52}{576} = \frac{13}{144}$

18. Given that $I(x) = \int \sqrt{\frac{x+1}{x}}\,dx$, where $I(9) = 3\sqrt{10} + \ell n\,(3 + \sqrt{10})$ then value of I(4) is

(1) $\ell n\left(\sqrt{2}+1\right)$ (2) $2\sqrt{5}+\ell n\left(2+\sqrt{5}\right)$ (3) $\sqrt{5}+\ell n\left(5\right)$ (4) none of these

Ans. **(2)**

Sol. Let $x = \tan^2\theta \Rightarrow dx = 2\tan\theta \, . \sec^2\theta \, d\theta$

$I = \int \frac{\sec\theta}{\tan\theta}\,(2\tan\theta \, . \sec^2\theta)\,d\theta = 2\int \sec^3\theta\, d\theta$

$I' = \int \sec\theta \sec^2\theta\, d\theta = \sec\theta \, . \tan\theta - \int \sec\theta \tan^2\theta d\theta$

$I' = \sec\theta \tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$

$2I' = \sec\theta\tan\theta + \int \sec\theta\ d\theta\ = I$

Thus $I = \sec\theta\tan\theta + \ell n\,|\sec\theta + \tan\theta| + c$

$I(x) = \sqrt{1+x}\ \ \sqrt{x}\ + \ell n\left|\sqrt{1+x}+\sqrt{x}\right| + c$

$I(9) = 3\sqrt{10}\ + \ell n\left|\sqrt{10}+3\right| + c = 3\sqrt{10}\ + \ell n\left(3+\sqrt{10}\right) \Rightarrow c = 0$

$I(x) = \sqrt{x(1+x)}\ + \ell n\left(\sqrt{1+x}+\sqrt{x}\right)$

$I(4) = 2\sqrt{5}\ + \ell n\left(2+\sqrt{5}\right)$

19. Let $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1/51 \\ 0 & 1 \end{bmatrix}$. Sum of all of elements of $\sum_{n=1}^{50} B^n$ is equal to

(1) 25 (2) 50 (3) 100 (4) 125

Ans. **(3)**

Sol. $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1/51 \\ 0 & 1 \end{bmatrix}$

let $P = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, $P^2 = -I$

$\Rightarrow B = P\,A(-P)$

$B^2 = P\,A(-P)P\,A(-P)\ = PA^2\,(-P)$

$B^3 = (P\,A^2 - P)(P\,A - P) = P\,A^3\,(-P)$

$\vdots$

$B^{50} = P\,A^{50}(-P)$

$\sum_{n=1}^{50} B^n\ = B + B^2 + \ldots\ldots + B^{50}$

$= P(A + A^2 + A^3 + ... + A^{50})(-P)$

$= P\begin{bmatrix} 50 & \frac{1}{51}+\frac{2}{51}+.......+\frac{50}{51} \\ 0 & 50 \end{bmatrix}(-P)$

$= P\begin{bmatrix} 50 & 25 \\ 0 & 50 \end{bmatrix}(-P)$

$= 25\left(P\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}(-P)\right)$

$= 25\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = 25\begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow$ sum of all element = 100

20. Two dice are rolled simultaneously. One die shows α and other die shows β. If the variance of $\alpha - \beta$ is $\frac{p}{q}$ where p and q are relatively prime then the sum of possible divisors of p is

(1) 50 (2) 45 (3) 48 (4) 52

Ans. **(3)**

Sol.

$\alpha-\beta$	case	p
5	(6,1)	1/36
4	(6,2),(5,1)	2/36
3	(6,3),(5,2)(4,1)	3/36
2	(6,4),(5,3)(4,2)(3,1)	4/36
1	(6,5),(5,4),(4,3)(3,2),(2,1)	5/36
0	(6,6),(5,5),(4,4),(3,3),(2,2),(1,1)	6/36
−1		5/36
−2		4/36
−3		3/36
−4		2/36
−5		1/36

$E(x^2) = \Sigma x^2 p(x) = 2\left[\frac{25}{36}+\frac{32}{36}+\frac{27}{36}+\frac{16}{36}+\frac{5}{36}\right] = \frac{105}{18}$

$\mu = E(X) = 0$ as data is symmetric

$\sigma^2 = E(X^2) = \Sigma x^2 p(x) = \frac{105}{18} = \frac{35}{6} \Rightarrow p = 35$

$p = 5 \times 7$

Sum of divisors is $(5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$

21. If A, B, C are angles of triangle and cos A + 2cos B + cos C = 2, a = 4, c = 7 then the value of cosA – cos C is equal to

(1) $\frac{99}{112}$ (2) $\frac{112}{99}$ (3) $\frac{33}{112}$ (4) $\frac{11}{112}$

Ans. **(1)**

Sol. $\frac{b^2+c^2-a^2}{2bc} + 2\left(\frac{a^2+c^2-b^2}{2ac}\right) + \frac{a^2+b^2-c^2}{2ab} = 2$

$\frac{b^2+49-16}{14b} + \frac{(16+49-b^2)}{28} + \frac{16+b^2-49}{8b} = 2$

$2b^3 - 11b^2 - 18b + 99 = 0 \Rightarrow b = \frac{11}{2}, -3, 3$

b = – 3, 3 (rejected)

$b = \frac{11}{2}$

$\cos A - \cos C = \frac{b^2+c^2-a^2}{2bc} - \frac{a^2+b^2-c^2}{2ab}$

$\frac{b^2+33}{77} - \frac{b^2-33}{44} = \frac{\frac{121}{4}+33}{77} - \frac{\frac{121}{4}-33}{44} = \frac{99}{112}$

22. If $f(x) = \left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}$ and local maximum value of f(x) is k then $\left(\frac{k}{e}\right)^8 + \left(\frac{k^8}{e^5}\right) + k^8$ is equal to

(1) $\frac{1}{e^2}+\frac{1}{e^5}+e^3$ (2) $\frac{1}{e^4}+\frac{1}{e^5}+e^3$ (3) $\frac{1}{e^3}+e^2+\frac{1}{e^5}$ (4) $\frac{1}{e^3}+\frac{1}{e^2}+e^5$

Ans. **(1)**

Sol. Let $\frac{2\sin x}{\sqrt{3e}} = \frac{1}{t}$

$y = (t)^{\frac{3e}{4t^2}}$

$\ell ny = \frac{3e}{4t^2}\ell n\, t$

$\frac{1}{y}.y' = \frac{3e}{4}\left[\frac{t-2t\ell n\, t}{t^4}\right]$

$y' = y\left(\frac{3e}{4}\left(\frac{1-2\ell n\, t}{t^3}\right)\right) = 0$

$\ell nt = \frac{1}{2}$

$t = e^{1/2}$

$y = (e^{1/2})^{\frac{3e}{4e}} = e^{3/8} = k \Rightarrow k^8 = e^3$

$\frac{e^3}{e^8} + \frac{e^3}{e^5} + e^3 = e^{-5} + e^{-2} + e^3$

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