

## QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

13 APRIL, 2023

9:00 AM to 12:00 Noon

Duration : 3 Hours

Maximum Marks : 300

## SUBJECT - MATHEMATICS

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**5**

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Roll No. : 20771637  
JEE Adv. 2022



AIR  
**15**

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## MATHEMATICS

1. The value of  $\int_0^{\infty} \frac{6dx}{e^{3x} + 6e^{2x} + 11e^x + 6}$  equal to  
 (1)  $\ln\left(\frac{32}{9}\right)$       (2)  $\ln\left(\frac{9}{8}\right)$       (3)  $\ln\left(\frac{8}{27}\right)$       (4)  $\ln\left(\frac{32}{27}\right)$

**Ans.** (4)

**Sol.** Let  $e^x = t$   
 $e^x dx = dt$

$$\begin{aligned} I &= \int_1^{\infty} \frac{\frac{6}{t} dt}{t^3 + 6t^2 + 11t + 6} = \int_1^{\infty} \frac{6}{t(t+1)(t+2)(t+3)} dt \\ &= \int_1^{\infty} \left( \frac{1}{t} - \frac{3}{t+1} + \frac{3}{t+2} - \frac{1}{t+3} \right) dt \\ &= \left\{ \ell n t - 3\ell n(t+1) + 3\ell n(t+2) - \ell n(t+3) \right\}_1^{\infty} \\ &= \left\{ \ell n \left\{ \frac{t(t+2)^3}{(t+1)^3(t+3)} \right\} \right\}_1^{\infty} = \ell n(1) - \ell n \frac{27}{32} = -\ell n \frac{27}{32} \end{aligned}$$

2. Consider the functional equation  $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$  then  $|f(3) - f'\left(\frac{1}{4}\right)|$  is equal to  
 (1) 5      (2) -3      (3) 0      (4) 7

**Ans.** (4)

**Sol.**  $x \rightarrow \frac{1}{x}$

$$3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$$

$$3f\left(\frac{1}{x}\right) + 2f(x) = x - 10$$

$$\frac{3}{2} \left[ \frac{1}{x} - 10 - 3f(x) \right] + 2f(x) = x - 10$$

$$\Rightarrow \frac{3}{2x} - 15 - \frac{9}{2}f(x) + 2f(x) = x - 10 \Rightarrow \frac{-5}{2}f(x) = x - \frac{3}{2x} + 5$$

$$f(x) = -\frac{2}{5}x + \frac{3}{5x} - 2$$

$$f(3) = -\frac{6}{5} + \frac{1}{5} - 2 = -3$$

$$f'(x) = \frac{-2}{5} - \frac{3}{5x^2} \Rightarrow f'\left(\frac{1}{4}\right) = \frac{-2}{5} - \frac{48}{5} = -10$$

$$\left| f(3) - f'\left(\frac{1}{4}\right) \right| = |-3 + 10| = 7$$

3. Let  $y_1(x)$  and  $y_2(x)$  satisfy the differential equation  $\frac{dy}{dx} = y + 7$ . If  $y_1(0) = 0$  &  $y_2(0) = 1$ , find the number of intersection of  $y_1(x)$  &  $y_2(x)$

(1)

**Sol.**  $\int \frac{dy}{y+7} = \int dx \Rightarrow |y+7| = ke^x$

$$\Rightarrow v_1 + 7 = k_1 e^x \text{ & } v_2 + 7 = k_2 e^x$$

$$v_1(0) = 0 \Rightarrow k_1 = 7 \text{ and } v_2(0) = 1 \Rightarrow k_2 = 8$$

$$\therefore y_1(x) = 7(e^x - 1) \text{ & } y_2(x) = 8e^x - 7$$

$$\text{Intersection } 7e^x - 7 = 8e^x - 7$$

$$\Rightarrow e^x = 0 \Rightarrow x \in \emptyset$$

$\therefore$  No point of intersection

4. If PQ is a focal chord of  $y^2 = 36x$  with  $PQ = 100$  and M divides PQ in the ratio 3 : 1, then the line through M perpendicular to PQ pass through -

(1) (1, 0)

$$\text{Ans. } (3)$$

t = 3

$\rightarrow P(81-54) \& O(1-6)$

M(21,9)

$$\Rightarrow L \text{ is } (y - 9) = \frac{-4}{3}(x - 21)$$

$$3y - 27 = -4x + 84$$

$$4x + 3y = 111$$

- 5.** Number of symmetric matrices of order  $3 \times 3$  which can be constructed from the elements  $\{0, 1, 2, 3, \dots, 9\}$  is

(1)

$$\begin{array}{ll} \text{Ans. } & (4) \\ \text{Sol. } & \left[ \begin{array}{ccc} x & a & b \\ a & x & c \\ b & c & x \end{array} \right] \end{array}$$

a can be filled in 10 ways

b can be filled in 10 ways

c can be filled in 10 ways

elements of diagonal can be filled in  $10^3$  ways

$$\text{total no of matrices} = 10^3 \times 10^3 = 10^6$$

6. Let the frequency distribution is

$x_i$	1	3	5	7	9
$f_i$	4	24	28	$\alpha$	8

If mean of observation is 5 then find  $\frac{3\alpha}{\text{Mean deviation about mean} + \text{variance}}$

(1) 8

(2) 7

(3) 4

(4) 6

**Ans.** (1)

**Sol.**  $5 = \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4+72+140+7\alpha+72}{64+\alpha}$

$$\Rightarrow 320 + 5\alpha = 288 + 7\alpha \Rightarrow 2\alpha = 32 \Rightarrow \alpha = 16$$

$$\text{M.D.}(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} \text{ where } \sum f_i = 64 + 16 = 80$$

$$\text{M.D.}(\bar{x}) = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80}$$

$$= \frac{128}{80} = \frac{8}{5}$$

$$\text{variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$= \frac{4 \times 16 + 24 \times 4 + 0 + 16 \times 4 + 8 \times 16}{80} = \frac{352}{80}$$

$$\frac{3\alpha}{\text{Mean deviation about mean} + \text{variance}} = \frac{3 \times 16}{\frac{128}{80} + \frac{352}{80}} = \frac{3 \times 16 \times 80}{480} = \frac{3 \times 8}{3} = 8$$

7. Let  $S_1 : \lim_{n \rightarrow \infty} \frac{2+4+6+\dots+2n}{n^2} = \frac{1}{2}$

$$S_2 : \lim_{n \rightarrow \infty} \frac{1^{15} + 2^{15} + \dots + n^{15}}{n^{16}} = \frac{1}{16}$$

(1)  $S_1$  is false,  $S_2$  is true

(2)  $S_1$  is true,  $S_2$  is false

(3) Both are true

(4) Both are false

**Ans.** (1)

**Sol.**  $S_1 : \lim_{n \rightarrow \infty} 2 \cdot \frac{n(n+1)}{2 \cdot n^2} = 1$

$$S_2 : \lim_{n \rightarrow \infty} \frac{\left( \frac{n^{16}}{16} + \dots \right)}{n^{16}} = \frac{1}{16}$$

8.  $2.2^2 - 3^2 + 2.4^2 - 5^2 + \dots$  (upto 20 terms) is equal to

**Ans.** (1310)

$$\begin{aligned}
 \textbf{Sol.} \quad & (2^2 + 4^2 + \dots + 20^2) + (2^2 - 3^2 + 4^2 - 5^2 + \dots + 20^2 - 21^2) \\
 &= 2^2(1^2 + 2^2 + \dots + 10^2) - (2 + 3 + 4 + \dots + 21) \\
 &= 4 \cdot \frac{10 \cdot 11 \cdot 21}{6} - \left( \frac{21 \cdot 22}{2} - 1 \right) \\
 &\equiv 1310
 \end{aligned}$$

9. Let  $g(x) = \sqrt{x+1}$  and  $f(g(x)) = 3 - \sqrt{x+1}$ , then find  $f(0)$

- (1) 3      (2) 2      (3) -1      (4) 0

**Ans.** (1)

**Sol.**     $g(x) = 0 \Rightarrow x = -1$

$$f(g(-1)) = 3 - \sqrt{-1+1} = 3$$

$$\Rightarrow f(0) = 3$$

- 10.** The number of seven digit numbers formed using 1, 2, 3, 4 whose sum of digits is 12, is



**Ans. (2)**

**Sol.** digits used (4, 3, 1, 1, 1, 1, 1)

digits used (4, 2, 2, 1, 1, 1, 1)

digits used (3, 3, 2, 1, 1, 1, 1)

digits used (3, 2, 2, 2, 1, 1,1)

digits used (2, 2, 2, 2, 2, 1,1)

$$\text{Total ways} = \frac{7!}{5!} + \frac{7!}{2!4!} + \frac{7!}{2!4!} + \frac{7!}{3!3!} + \frac{7!}{5!2!}$$

$$= 42 + 105 + 105 + 140 + 21 = 413$$

Alter

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12$$

$$x_j \in \{1, 2, 3, 4\}$$

$$\text{coeff. of } x^{12} \text{ in } (x + x^2 + x^3 + x^4)^7 = x^7(1 - x^4)^7(1 - x)^{-7}$$

coeff. of  $x^5$  in  $(1 - x^4)^7(1 - x)^{-7}$

$$= {}^7C_0 {}^{11}C_5 - {}^7C_1 {}^7C_1 \equiv 413$$

- 11.** Let 1, 2, 3, ..... 10 are first terms of 10 A.P.s respectively. If the common difference of these 10 AP is are 1, 3, 5, 7, ..... respectively then find  $\sum_{i=1}^{10} S_i$  where  $S_i$  represents the sum of first 12 terms of  $i^{\text{th}}$  A.P.

- (1) 7260                          (2) 7240                          (3) 7230                          (4) 7220

**Ans.** (1)

**Sol.**  $S_i = \frac{12}{2} [2i + 11 \times (2i - 1)] = 6[24i - 11]$

$$\sum_{i=1}^{10} S_i = 6 \sum_{i=1}^{10} (24i - 11) = 6 \left[ \frac{24 \times 10 \times 11}{2} - 110 \right] \\ = 6[1320 - 110] = 1210 \times 6 = 7260$$

- 12.** Find  $\left\{ \frac{4^{2022}}{15} \right\}$  { . } is fraction part function

- (1)  $\frac{1}{15}$       (2)  $\frac{4}{15}$       (3)  $\frac{7}{15}$       (4)  $\frac{1}{15}$

**Ans.** (1)

**Sol.**  $4^{2022} = 16^{1011} = (15 + 1)^{1011} = 15\lambda + 1$

$$\Rightarrow \left\{ \frac{4^{2022}}{15} \right\} = \frac{1}{15}$$

- 13.** Consider the binomial expansion  $\left( \sqrt{x} - \frac{6}{x^{3/2}} \right)^n$  where  $n \leq 15$ . If the coefficient of term independent of  $x$  is  $\alpha$  and sum of coefficients of all terms except the coefficient of independent term is 649 then find  $\lambda$  where coefficient of  $x^{-n}$  is  $24\lambda$

- (1) -36      (2) -18      (3) -9      (4) -6

**Ans.** (1)

**Sol.**  $T_{r+1} = {}^n C_r (x)^{\frac{n-r}{2}} (-6)^r x^{-3r/2}$

$$= {}^n C_r (x)^{\frac{n-4r}{2}} (-6)^r$$

$$\frac{n-4r}{2} = 0 \Rightarrow r = \frac{n}{4}$$

$$x = 1 \Rightarrow (-5)^n$$

$$\text{So } (-5)^n - {}^n C_{n/4} (-6)^{n/4} = 649 \Rightarrow n = 4$$

$$\text{Now } \frac{n-4r}{2} = -n \Rightarrow 3n = 4r \Rightarrow r = 3$$

so coefficient of  $x^{-n}$  is  ${}^4 C_3 (-6)^3 = 4 \times -216 = 24\lambda$

$$\lambda = -36$$

- 14.** Let  $f(x) = x - \sin 2x + \frac{\sin 3x}{3}$  defined on  $[0, \pi]$  then the maximum value of  $f(x)$  is.

(1)  $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{3}$

(2)  $\frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{2}{3}$

(3)  $\frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$

(4)  $\pi + \frac{\sqrt{3}}{2} + \frac{1}{3}$

**Ans.** (3)

Sol.  $f'(x) = 1 - 2\cos(2x) + \cos(3x) = 0$

$$\cos x = t \Rightarrow 4t^3 - 3t - 2(2t^2 - 1) + 1 = 0$$

$$4t^3 - 4t^2 - 3t + 3 = 0$$

$$4t^2(t-1) - 3(t-1) = 0 \Rightarrow t = 1, t = \pm \frac{\sqrt{3}}{2}$$

$$\cos x = 1 \Rightarrow x = 0, \cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$f''(x) = 4\sin(2x) - 3\sin(3x)$$

$$f''\left(\frac{\pi}{6}\right) = 4 \times \frac{\sqrt{3}}{2} - 3 = 2\sqrt{3} - 3 > 0$$

$$f''\left(\frac{5\pi}{6}\right) = 4 \times \left(-\frac{\sqrt{3}}{2}\right) - 3 = -2\sqrt{3} - 3 < 0$$

$$f(0) = 0, f(\pi) = \pi, f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{3}$$

$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3} \rightarrow \text{Maximum as } f''\left(\frac{5\pi}{6}\right) < 0$$

15. Let  $y = f(x) = \max\{\sin x, \cos x\}$ ,  $-\pi \leq x \leq \pi$  find the area bounded by  $y = f(x)$  and the x-axis

(1) 8

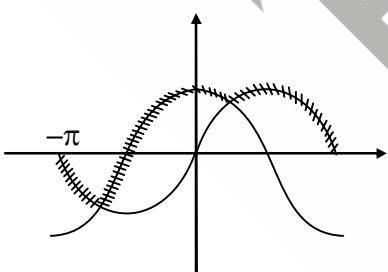
(2) 6

(3) 4

(4)  $\frac{4}{3}$

Ans. (3)

Sol.



$$\int_{-\pi}^{-3\pi/4} (-\sin x) dx + \int_{-3\pi/4}^{-\pi/2} (-\cos x) dx + \int_{-\pi/2}^{\pi/4} \cos x dx + \int_{\pi/4}^{\pi} \sin x dx$$

$$= (\cos x)_{-\pi}^{-3\pi/4} dx - (\sin x)_{-3\pi/4}^{-\pi/2} + (\sin x)_{-\pi/2}^{\pi/4} - (\cos x)_{\pi/4}^{\pi}$$

$$= \left(-\frac{1}{\sqrt{2}} + 1\right) - \left(-1 + \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} + 1\right) - \left(-1 - \frac{1}{\sqrt{2}}\right)$$

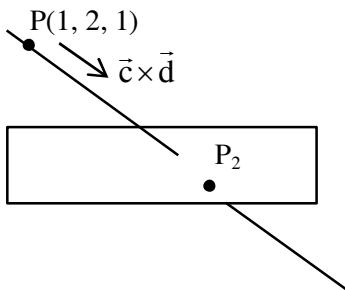
$$= 4$$

16. Find the distance of the point  $P(1, 2, 1)$  from the plane  $x + y + z = 6$  measured along the line of shortest distance for the lines  $\frac{x-1}{1} = \frac{y}{2} = \frac{z}{1}$  and  $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$
- (1)  $4\sqrt{3}$       (2)  $2\sqrt{3}$       (3)  $3\sqrt{2}$       (4)  $4\sqrt{2}$

**Ans.** (2)

**Sol.**  $\vec{c} \times \vec{d} = \langle 4, -4, 4 \rangle$

$$L \text{ is } \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$$



$$P_2(\lambda + 1, -\lambda + 2, \lambda + 1)$$

$$x + y + z = 6 \Rightarrow \lambda = 2$$

$$\therefore P_2(3, 0, 3)$$

$$\Rightarrow PP_2 = \sqrt{4+4+4} = 2\sqrt{3}$$

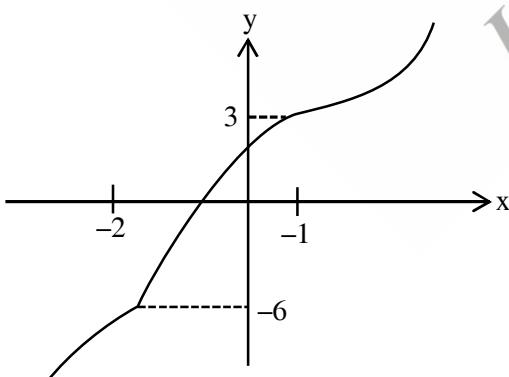
17. The values of  $\alpha$  so that equation  $x|x - 1| + |x + 2| + \alpha = 0$  have exactly one solution
- (1)  $(-\infty, \infty)$       (2)  $(-6, 3)$       (3)  $(3, \infty)$       (4)  $(-\infty, -6)$

**Ans.** (1)

**Sol.**  $x|x - 1| + |x + 2| = -\alpha$

$$\text{let } y = x|x - 1| + |x + 2|$$

$$y = \begin{cases} x^2 + 2 & ; \quad x \geq 1 \\ -x^2 + 2x + 2 & ; \quad -2 \leq x < 1 \\ -x^2 - 2 & ; \quad x < -2 \end{cases}$$



as  $y \in \mathbb{R}$

hence

$$-\alpha \in (-\infty, \infty)$$

18. Let probability of getting head is thrice of probability of getting tail. A coin is tossed until one head or 3 tails are obtained. If number of trials is 24 then the mean of experiment is  
(1) 28.5                    (2) 31.5                    (3) 25                    (4) 26.5

**Ans. (2)**

**Sol.**    **Case-I**               $H \rightarrow 1 \times \frac{3}{4}$

$$\text{Case-II} \quad \text{TH} \rightarrow 2 \times \frac{1}{4} \times \frac{3}{4}$$

$$\text{Case-III} \quad TTH \rightarrow 3 \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}$$

$$\text{Case-IV} \quad \text{TTT} \rightarrow 3 \times \left(\frac{1}{4}\right)^3$$

$$\text{Mean} = \left( \frac{3}{4} + \frac{6}{16} + \frac{9}{64} + \frac{3}{64} \right) \times 24$$

$$= 31.5$$

- 19.** Let  $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ ,  $\vec{b} = 3\hat{i} + 3\hat{j} + 7\hat{k}$ ,  $\vec{c} = 7\hat{i} + 8\hat{j} + 9\hat{k}$ . If  $\vec{a} \times \vec{b} = \vec{c} + \vec{d}$  then  $|\vec{d}|$  is  
 (1)  $\sqrt{194}$       (2)  $\sqrt{190}$       (3)  $\sqrt{187}$       (4)  $\sqrt{185}$

**Ans.** (1)

$$\text{Sol. } \vec{d} = \vec{a} \times \vec{b} - \vec{c}$$

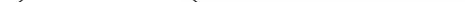
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & 3 & 7 \end{vmatrix} = 6\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{d} = (6\hat{i} + \hat{j} - 3\hat{k}) - (7\hat{i} + 8\hat{j} + 9\hat{k}) = -\hat{i} - 7\hat{j} - 12\hat{k}$$

$$|\vec{d}| = \sqrt{1 + 49 + 144} = \sqrt{194}$$

20. If  $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$  (for  $x \geq 0$ ) then  $\sin\left((x^2+x+5)\frac{\pi}{2}\right) - \cos((x^2+x+5)\pi)$  is  
 (1) 1      (2) 2      (3) 3      (4) 4

**Ans. (3)**

(3) 

$$\sin^{-1} \left( \frac{x+1}{\sqrt{x^2 + 2x + 2}} \right) = \tan^{-1} x$$

$$\Rightarrow \tan^{-1}(x+1) - \tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{1}{1+x(x+1)}\right) = \frac{\pi}{4}$$

$$\Rightarrow 1 + x(x+1) = 1 \Rightarrow x = 0, x = -1$$

$$\text{so } \sin\left(\frac{5\pi}{2}\right) - \cos(5\pi) = 1 - (-1) = 2$$

- 21.** If  $\frac{dy}{dx} = 6e^x + e^{2x} + e^{3x}$ , then  $y(2) - y(0)$  is

(1)  $e^2 + \frac{6e^4}{4} - \frac{e^6}{3} + \frac{15}{6}$

(2)  $6e^2 + \frac{e^4}{3} + \frac{e^6}{2} - \frac{15}{6}$

(3)  $6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{41}{6}$

(4)  $e^2 + \frac{6e^4}{2} + \frac{e^6}{3} - \frac{15}{6}$

**Ans.** (3)

**Sol.**  $y = 6 \int e^x dx + \int e^{2x} dx + \int e^{3x} dx$

$$y = 6e^x + \frac{e^{2x}}{2} + \frac{e^{3x}}{3} + c$$

$$y(2) = 6e^2 + \frac{e^4}{2} + \frac{e^6}{3}, y(0) = 6 + \frac{1}{2} + \frac{1}{3} = \frac{41}{6}$$

$$y(2) - y(0) = 6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{41}{6}$$

- 22.** Plane  $P_3$  is passing through the point  $(1, 1, 1)$  and line of intersection of  $P_1$  and  $P_2$  where  $P_1 : 2x - y + z = 5$  and  $P_2 : x + 3y + 2z + 2 = 0$ , then distance of  $(1, 1, 10)$  from  $P_3$  is

(1)  $\frac{126}{\sqrt{558}}$

(2)  $\frac{63}{\sqrt{558}}$

(3)  $\frac{252}{\sqrt{558}}$

(4) None of these

**Ans.** (1)

**Sol.**  $P_3$  is  $P_1 + \lambda P_2 = 0$

$$(2x - y + z - 5) + \lambda(x + 3y + 2z + 2) = 0$$

$$-3 + \lambda(8) = 0 \Rightarrow \lambda = 3/8$$

$$\therefore P_3 : 19x + y + 14z = 34$$

$$\text{distance} = \frac{|19+1+140-34|}{\sqrt{361+1+196}} = \frac{126}{\sqrt{558}}$$

- 23.** The negation of  $((A \wedge (B \vee C)) \rightarrow (B \wedge C)) \rightarrow A$  is equivalent to

(1)  $\sim B$

(2)  $\sim(A \wedge B)$

(3)  $\sim A$

(4)  $\sim(A \vee B)$

**Ans.** (3)

**Sol.**  $((A \wedge (B \vee C)) \rightarrow (B \wedge C)) \equiv \sim(A \wedge (B \vee C)) \vee (B \wedge C)$

$$\text{So } ((A \wedge (B \vee C)) \rightarrow (B \wedge C)) \rightarrow A$$

$$\equiv (\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \rightarrow A$$

$$\equiv \sim(\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \rightarrow A$$

$$\equiv \sim(\sim(\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \vee A)$$

$$\equiv \sim(A \wedge (B \vee C)) \wedge \sim(B \wedge C) \vee A$$

$$\equiv \sim A$$

24. Consider the system of linear equations

$$2x + 4y + bz = 2a$$

$$x + y + z = 6$$

$$x - y - z = 8$$

then the correct option are -

- (1) If  $a = 5$  and  $b = 4$  then system has unique solution
- (2) If  $b \neq 4$  then system has unique solution only for  $a = 1$
- (3) If  $a = 5$  and  $b = 4$  then system has infinite solution
- (4) If  $a = 5$  and  $b = 4$  then system has no solution

**Ans. (3)**

**Sol.**  $D = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 4 & b \\ 1 & 1 & 1 \\ 2 & 0 & 0 \end{vmatrix} = 2(4 - b)$

$$D_1 = \begin{vmatrix} 2a & 4 & b \\ 6 & 1 & 1 \\ 8 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2a & 4 & b \\ 6 & 1 & 1 \\ 14 & 0 & 0 \end{vmatrix} = 14(4 - b)$$

$$D_2 = \begin{vmatrix} 2 & 2a & b \\ 1 & 6 & 1 \\ 1 & 8 & -1 \end{vmatrix}$$

$$2[-14] - 2a(-2) + b(2) = 4a + 2b - 28$$

$$D_3 = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 1 & 6 \\ 1 & -1 & 8 \end{vmatrix} = 2[14] - 4(2) + 2a(-2)$$

$$= 28 - 8 - 4a = 20 - 4a$$



## SATYAM CHAKRAVORTY

(Classroom) →

selected for

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