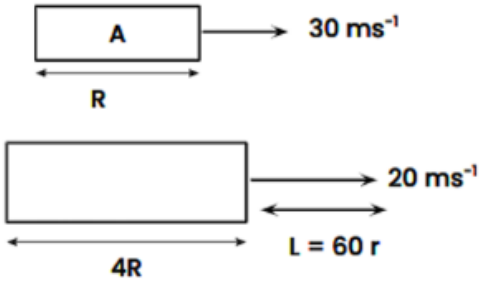
JEE-Mains-13-04-2023 [Memory Based] [Morning Shift]

Physics

Question: Train A takes 3.5 seconds less time than train B. Find L = ?



Answer: 1800.00

Solution:

$$\frac{64l}{20}-\frac{61l}{30}=35$$

$$\frac{l}{10}\left[32-\frac{61}{3}\right]=35$$

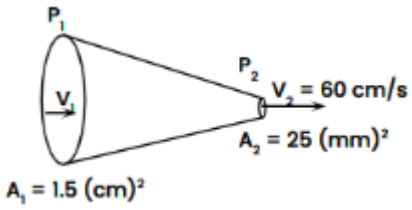
$$\frac{l}{10}\times\left[\frac{35}{3}\right]=35$$

$$l=30\text{ m}$$

$$L=60l$$

$$L=1800\text{ m}$$

Question: There is streamline flow of water in horizontal pipe find pressure difference at the cross sections ΔP.



Answer: 175.00

Solution:

$$P_1+\frac{1}{2}\rho V_1^2=P_2+\frac{1}{2}\rho V_2^2$$

$$P_1-P_2=\frac{\rho}{2}\left[V_2^2-V_1^2\right]$$

$$A_1V_1=A_2V_2$$

$$\left(1.5\times10^{-4}\right)\times V_1=25\times10^{-6}\times\frac{60}{100}$$

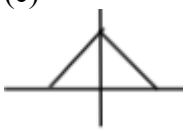
Question: A particle is performing SHM draw the graph of TE -PE vs position

Options:

(a)

(b)

(c)



(d)

Answer: (a)

Question: If the height of the tower used for L.D.S. is increased by 21% then percentage change in range is

Options:

(a) 10%

(b) 21%

(c) 19%

(d) 42%

Answer: (a)

Solution:

$R = \sqrt{2h_T R_e}$

$R' = \sqrt{2(1.21)h_T R_e}$

$R' = 1.1R$

$\frac{0.1}{1} \times 100 = 10\%$

Question: A dipole of charge 0.01 C and separation 0.4 mm, is placed in an electric field of strength 10 dyne/C, Find the maximum torque exerted on the dipole in the field

Options:

(a) 4×10^{-9} Nm

(b) 2×10^{-10} Nm

(c) 4×10^{-10} Nm

(d) 2×10^{-9} Nm

Answer: (c)

Solution:

$\vec{\tau} = \vec{p} \times \vec{E}$

$\tau_{max} = pE = qlE$

$= 0.01 \times 0.4 \times 10^{-3} \times 10 \times 10^{-5}$

$= 4 \times 10^{-10}$ Nm

Question: Two bodies having same linear momentum have ratio of kinetic energy as 16 : 9. Find the ratio of masses of these bodies.

Options:

(a) 9/16

(b) 4/3

(c) 3/4

(d) 16/9

Answer: (a)

Solution:

$K = \frac{P^2}{2m}$

$m \propto \frac{1}{K}$

Question: What is center of gravity of semi-circular disc of radius (R)?

Options:

(a) 2R/π

(b) 4R/3π

(c) R/2

(d) 3R/8

Answer: (b)

Solution: 4R/3π

Question: Pressure for polytropic process P varies with volume V as P = av^{-3}, find out the bulk modulus.

Options:

(a) 3V

(b) 3P

(c) P

(d) V

Answer: (b)

Solution:

$B = \gamma P$

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{B}{\rho}} \quad \left(PV^{\gamma} = h\right)$

$B = xP = 3P$

Question: The work function for two metals are 9 eV and 4.5 eV. Find the approx. difference between their threshold wavelength. (use hc = 1240 eV - nm)

Options:

(a) 138 nm

(b) 130 nm

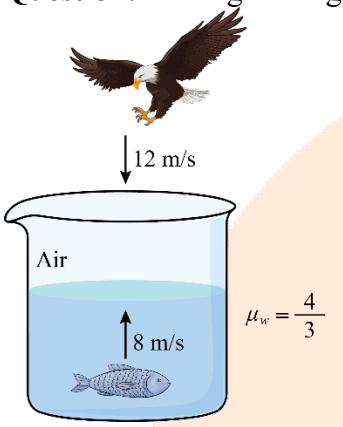
(c) 112 nm
(d) 145 nm
Answer: (a)
Solution:

$$\lambda_0 = \frac{hc}{\phi}$$

$$\lambda_2 - \lambda_1 = \left[\frac{1240}{4.5} - \frac{1240}{9}\right] \text{nm}$$

=138 nm

Question: In the given figure, find the speed of bird as seen by fish



Options:
(a) 24 m/s
(b) 16 m/s
(c) 20 m/s
(d) 12 m/s
Answer: (b)
Solution:

$$\frac{AD}{RD} = \frac{K_2}{K_1}$$

$$AD = \frac{K_2}{\mu_1} \times RD$$

$$v_b = \frac{4}{3} \times 12 = 16 \text{ m/s}$$

Question: If a wire of resistance R is connected across V_0, then power is P_0. The wire is cut into two equal parts and connected with V_0 individually, then sum of power dissipated is P_1, then P_0/P_1 is 1/x. Find the value of *x*.
Answer: 4.00
Solution:

$$P_0 = \frac{V_0^2}{R}$$

$$P_1 = \frac{V_0^2}{R/2} + \frac{V_0^2}{R/2} = \frac{V_0^2}{R/2} = \frac{4V_0^2}{R}$$

$x = 4$

Question: A particle is performing SHM having position x = A cos 30°, and A = 40 cm. If its kinetic energy at this position is 200 J, the value of force constant in $\left(\frac{kN}{m}\right)$ is

Answer: 10.00

Solution:

$$\frac{1}{2}mu^2 = \frac{1}{2}mw^2\left(A^2 - x^2\right) = 200$$

$$= \frac{1}{2}k\left[A^2 \cdot x^2\right] = 200$$

$$= \frac{1}{2}k\left[A^2 - \frac{3A^2}{4}\right] = 400$$

$$= k\frac{A^2}{4} = 400$$

$$l = \frac{400 \times 4}{0.16}$$

Question: For the given radioactive decay ${}^{298}_{94}X \rightarrow {}^{294}_{92}Y + {}^{4}_{2}\alpha + Q$ value binding energy per nucleon of X, Y and α are a, b and c. The Q value is equal to

Options:

(a) 294b + 4c – 298a
(b) 92b + 2c – 94a
(c) 294b + 4c + 298a
(d) 92b + 2c + 94a

Answer: (a)

Solution:

$$Q = U_i - U_f$$

$$Q = \left(BE\right)_f - \left(BE\right)_i$$

Question: The energy of He^+ in 2nd orbit is –13.6 eV than energy of Be^{+++} in n = 4

Options:

(a) –3.4 eV
(b) –27.2 eV
(c) –13.6 eV
(d) –54.4 eV

Answer: (c)

Solution:

$$E = -13.6eV\frac{Z^2}{n^2} = 13.6eV \times \frac{16}{16}$$

Question: Which of the following shows time varying magnetic field.

Options:

(a) Constant electric field
(b) Antenna signal
(c) Permanent magnet
(d) Linearly varying electric field

Answer: (b)
Solution: Antenna signal

Question: Solid sphere rolls on a horizontal plane. Ratio of angular momentum about COM to total energy is π/22. Find ω?
Answer: 4.00
Solution:

$$\frac{L_{com}}{K} = I\text{cm}\omega = \frac{2}{5}mr^2\omega$$

$$K = \frac{1}{2}\times\frac{2}{5}mr^2\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}\left[\frac{7}{5}mr^2\right]\omega^2$$

$$\frac{2}{7\omega}\times 2 = \frac{\pi}{22}$$

$$\omega = \frac{4\times 22}{7\times 22/7} = 4$$

Question: If $m = 5 \pm 0.2$ and $v = 20 \pm 0.4,$ calculate percentage error in measurement of KE.
Solution:

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow \left[\frac{\Delta K}{v}\right]\times 100 = \left[\left|\frac{\Delta m}{m}\right| + \left|2\frac{\Delta v}{v}\right|\right]\times 100$$

$$= \left[\frac{0.2}{5} + 2\times\frac{0.4}{20}\right]\times 100$$

Question: A bullet of mass 10 g is fixed with a velocity of 600 m/s mass of gun is 3 kg and length if gun is 10 cm. The impulse on the gun is
Options:
(a) 6
(b) 12
(c) 36
(d) 8
Answer: (a)
Solution:

$$I = \Delta p$$

$$M_2V_2 = M_1V_1$$

$$= \frac{10}{1000}\times 600 = 6$$

Question: Two bodies having same linear momentum and ratio of kinetic energy as 16 : 9. Find the ratio of masses of these bodies.
Options:
(a) 9/16
(b) 4/3

(c) 3/4
(d) 16/9
Answer: (a)

Solution: $K = \frac{p^2}{2m} \propto \frac{1}{m}$

Question: In a polytropic process pressure of a gas varies with volume as $P = \alpha V^{-3}$ find bulk modulus.
Answer: 3.00

Solution: $V_{sound} = \sqrt{\frac{B}{\rho}}$

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Chemistry

Question: Which of the following is incorrect about Borazine?
Options:
(a) It reacts with water.
(b) It resembles with benzene.
(c) It contain banana bond.
(d) It has cyclic structure
Answer: (c)
Solution: $B_3N_3H_6$ is inorganic benzene

Question: $A_2 + B_2 \rightarrow 2AB$; $\Delta H = 200$ KJ/mol
Ratio of Bond enthalpy of $A_2 : B_2 : AB = 1:1.5:1$
Find bond enthalpy of B_2.
Options:
(a) 400
(b) 200
(c) 100
(d) 250
Answer: (a)
Solution:

$$\underset{1}{A_2} + \underset{1.5}{B_2} \rightarrow \underset{2}{2AB}$$

$x + 1.5x - 2x = 400$

$3.5x - 2x = 400$

$$x = \frac{400}{1.5} = 266.6$$

$B_2 = 1.5x = 1.5 \times 266.6 = 400$

Question: Radius of 2nd orbit of He^+ is r_0. Radius of 4th orbit of Be^{3+} is xr_0. Find x
Options:
(a) 2
(b) 4
(c) 1
(d) 3
Answer: (a)
Solution: $r_{He^+} = 2r_0$, $r_{Be^{+3}} = r_0 \times 4$

Question: Which of the following free radical helps in depletion of ozone layer?
Options:
(a) NO•
(b) Cl•
(c) OH•
(d) CH_3•
Answer: (b)
Solution: Fact based.

Question: Which if the following is incorrect match?
Options:
(a) Zinc- Liquation
(b) Cu-Electrolysis
(c) Ni-Mond Process
(d) Ti-Van Arkel method
Answer: (a)
Solution: Fact based.

Question: In which of the following options the species changes from paramagnetic to diamagnetic and bond order increases.
Options:
(a) $N_2 \rightarrow N_2^+$
(b) $O_2 \rightarrow O_2^-$
(c) $NO \rightarrow NO^+$
(d) $O_2 \rightarrow O_2^+$
Answer: (c)
Solution: NO has B.O = 2.5 and $NO^+ = 3$

Question: Match the following.

Column-I	**Column-II**
(A) Nylon-6	**(P)** Cross Linking
(B) Vulcanised Rubber	**(Q)** Caprolactum
(C) isoprene	**(R)** Synthetic Rubber
(D) Neoprene	**(S)** Chloroprene

Options:
(a) A – R; B – P; C – S; D – Q
(b) A – Q; B – P; C – R; D – S
(c) A – S; B – P; C – Q; D – R
(d) A – Q; B – P; C – S; D – R
Answer: (b)
Solution: Fact based

Question: When lyophilic sol is added to the lyophobic sol then?

Options:
(a) Fixed layer of lyophobic sol is formed over lyophilic sol
(b) Fixed layer of lyophilic sol is formed over lyophobic sol
(c) Emulsion
(d) Electrophoresis
Answer: (b)
Solution: Fact based

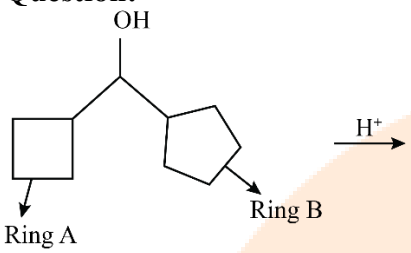
Question: Statement-1: Permutit process is more efficient than synthetic resin method.
Statement-2: In synthetic resin method, soluble salt of sodium is formed.
Options:
(a) statement 1 is correct and statement 2 is incorrect
(b) statement 1 is incorrect and statement 2 is correct
(c) both statement 1 and statement 2 are incorrect
(d) both statement 1 and statement 2 are correct
Answer: (c)
Solution: Fact based.

Question:
Statement-1: Ionisation enthalpy decrease down the group.
Statement-2: Electron gain enthalpy of F is more negative than Cl.
Statement-3: Al_2O_3 and NO are amphoteric oxides.
Statement-4: Electronegativity depends upon the number of surrounding atoms.
Number of incorrect statements are:
Options:
(a) 1
(b) 2
(c) 3
(d) 4
Answer: (c)
Solution: Fact based.

Question: Which of the following is incorrect match?
A) $Ag(CN)_2^-$ - Photography
B) EDTA- Hard water
C) Wilkinson catalyst- $[(Ph_3P)_3RhCl]$
D) Chlorophyll-Co
Options:
(a) Both A and D
(b) A, B, and C
(c) B and C
(d) A and C
Answer: (a)
Solution: Fact based

Question: ClF_5 at room temperature is
Options:
(a) Gas and square pyramidal structure
(b) Liquid and square pyramidal structure
(c) Gas and trigonal bipyramidal
(d) Liquid and trigonal bipyramidal
Answer: (b)
Solution: hybridization is sp^3d^2 and exists as liquid.

Question:

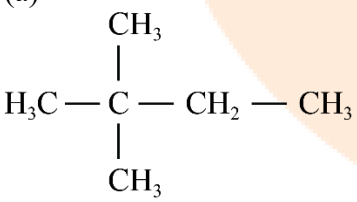


Options:
(a) Ring A is only 6 membered ring
(b) Two 5 membered rings are formed
(c) Two 6 membered rings or formed
(d) 7 member ring formation taken
Answer: (c)
Solution: Carbocation ring expansion occurs.

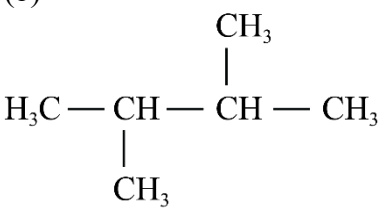
Question: What is the major product formed in the following reaction?

$$CH_3 - (CH_2)_4 - CH_3 \xrightarrow[HCl,\ heat]{Anhy.\ AlCl_3} \text{Major product}$$

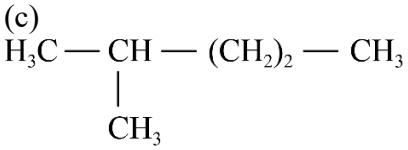
Options:
(a)



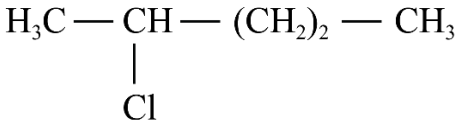
(b)



(c)



(d)



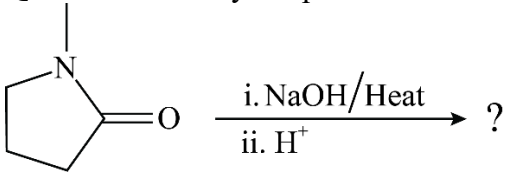
Answer: (c)
Solution: Isomerization reaction occurs.

Question: An organic compound on combustion gives 0.22 g of CO_2 and 0.126 g of H_2O. If the percentage of C in given organic compound is 40%, the percentage of H will be?
Options:
(a) 9.33%
(b) 14%
(c) 10%
(d) None
Answer: (a)
Solution: 5 milimoles of CO2 and 7milimoles of H2O is formed thus one mole has 5 carbon or 60 gm carbon , now since its 40% of organic compound therefore organic compound mass is 150 and this concludes mass % of H is 9.33

Question: For the 1st order reactions, the ratio of $t_{50\%}$ to $t_{87.5\%}$ will be
Options:
(a) 2
(b) 3
(c) 4
(d) same
Answer: (b)
Solution: Fact based

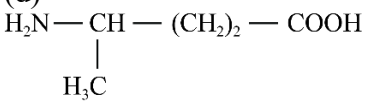
Question: The pair of lanthanides with exceptionally high 3rd ionisation enthalpy than neighbouring elements:
Options:
(a) Lu and Yb
(b) Eu and Gb
(c) Eu and Yb
(d) Dy and Yb
Answer: (c)
Solution: Fact based

Question: Identify the product formed in the following reaction



Options:
(a) $H_2N — (CH_2)_3 — CHO$

(b) H_3C — NH — $(CH_2)_3$ — CH_2OH
(c) H_3C — NH — $(CH_2)_3$ — COOH
(d)



Answer: (c)
Solution: nucleophilic addition reaction occurs

Question: Consider the given reaction:
$Be(OH)_2 + Sr(OH)_2 \rightarrow$ Product
Incorrect statement regarding the product is given as:
Options:
(a) Be is tetrahedrally bonded in the product
(b) Be forms cationic part
(c) It is an acid base reaction
(d) $Be(OH)_2$ acts as a Lewis acid
Answer: (b)
Solution: Be forms complex which is anionic part

Question: Match the column.

Column I	**Column II**
(A) Troposphere	**(P)** From 10 - 50 km from sea level
(B) Stratosphere	**(Q)** Uto 10 km from sea level
(C) Mesosphere	**(R)** From 85 km to ≈ 700 km from sea level
(D) Thermosphere	**(S)** From 50 km to 85 km from sea level

Options:
(a) A-P; B-Q; C-R; D-S
(b) A-R; B-S; C-P; D-Q
(c) A-Q; B-P; C-S; D-R
(d) A-S; B-R; C-P; D-Q
Answer: (c)
Solution: Fact based

Question: A solution is isotonic with glucose having concentration 0.05 M at a certain temperature. If the volume of the solution is 1 L, find the molar mass of the solute in g/mol if 12 g of solute is mixed to form the solution.

Options:
(a) 120
(b) 240
(c) 360
(d) None
Answer: (b)

Solution: $\pi_1 = \pi_2$

$0.05 = \frac{12}{\text{Mol. mass}}$ or mol. Mass = 240

Question: Glyceraldehyde $\xrightarrow[\text{II.}H_3O^+,\ \text{III.}HNO_3]{\text{I.HCN}}$ A + B

Then select the correct option about the product A and B.

Options:
(a) Both are optically active
(b) Both are optically inactive
(c) One is optically active and another is optically inactive
(d) None of these
Answer: (c)
Solution: Reaction based

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Mathematics

Question: For the differentiable function $f:R-\{0\}\to R$, let $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

Answer: 13.00

Solution:

$$\left[3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10\right]\times 3$$

$$\left[2f(x)+3f\left(\frac{1}{x}\right)=x-10\right]\times 2$$

$$5f(x)=\frac{3}{x}-2x-10$$

$$\Rightarrow f(x)=\frac{1}{5}\left(\frac{3}{x}-2x-10\right)$$

$$f'(x)=\frac{1}{5}\left(-\frac{3}{x^2}-2\right)$$

$$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=\left|\frac{1}{5}(1-6-10)+\frac{1}{5}(-48-2)\right|$$

$$=|-3-10|$$

$$=13$$

Question: How many symmetric matrices of order 3×3 can we form $(0,1,2,...,9)$ 3×3.

Answer: 10^6

Solution:

$$A=\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix},\ a,b,c,d,e,f\in\{0,1,2,....9\}$$

Number of matrices $=10^6$

Question: $S_1,S_2,...,S_{10}$, be sum of 12 terms of 10 APs with 1st terms: $1,2,3,....,10$ & common differences: $1,3,5,7,....,19$. Find $S_1+S_2+...+S_{10}$.

Answer: 7260.00
Solution:

$$S_k = 6\left(2k + (11)(2k-1)\right)$$

$$S_k = 6\left(2k + 22k - 11\right)$$

$$S_k = 144k - 66$$

$$\sum_{1}^{10} S_k = 144\sum_{k=1}^{10} k - 66\times 10$$

$$= 144\times\frac{10\times 11}{2} - 660$$

$$= 7920 - 660$$

$$= 7260$$

Question: A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occurs. If X denotes the number of t are of the coin, then the mean of X is

Answer: $\frac{21}{16}$

Solution:

$$P(H) = \frac{3}{4}$$

$$P(T) = \frac{1}{4}$$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4}\times\frac{3}{4}$	$\left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^2\times\frac{3}{4}$

Mean $\bar{X} = \frac{3}{4} + \frac{3}{8} + 3\left(\frac{1}{64} + \frac{3}{64}\right)$

$$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$= 3\left(\frac{7}{16}\right)$$

$$= \frac{21}{16}$$

Question: Number of 7 digit numbers using $\{1,2,3,4\}$ such that sum of all digits is 12 is _____.

Answer: 413.00
Solution:

$x_1+x_2+x_3+x_4+x_5+x_6+x_7=12,\ x_i\in\{1,2,3,4\}$

No. of solutions $={}^{5+7-1}C_{7-1}-\frac{7!}{6!}-\frac{7!}{5!}=413$

Question: $\int_0^{\infty}\frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx=$

Answer: $\ln\left(\frac{32}{27}\right)$

Solution:

Given, $\int_0^{\infty}\frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx$

Put $e^x=t\Rightarrow e^x dx=dt$

$\int_1^{\infty}\left(\frac{6}{t^3+6t^2+11t+6}\right)\frac{dt}{t}$

$=\int_1^{\infty}\frac{6}{t(t+1)(t+2)(t+2)}dt$

$=\int_1^{\infty}3\left(\frac{1}{t(t+3)}-\frac{1}{(t+1)(t+2)}\right)$

$=\int_1^{\infty}\frac{1}{t}-\frac{1}{t+3}-\frac{3}{t+1}+\frac{3}{t+2}$

$\ln\frac{t\times(t+2)^3}{(t+3)(t+1)^3}\Big|_1^{\infty}$

$\ln 1-\ln\frac{27}{32}=\ln\left(\frac{32}{27}\right)$

Question: Let the tangent and normal at the point $\left(3\sqrt{3},1\right)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangent at the points P and Q on the circle intersects at the point (α,β), then $\alpha^2-\beta^2$ is equal to

Answer: 60.8

Solution:

Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$

$y=4$

$$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$$

$$y=-8$$

$$x^2+(y-4)(y+8)=0$$

$$x^2y^2+4y-32=0$$

$$hx+ky+2(y+k)-32=0$$

$$k=-2$$

$$hx+2k-32=0$$

$$hx=36$$

$$\alpha=h=\frac{36}{2\sqrt{5}}$$

$$\beta=k=-2$$

$$\alpha^2-\beta^2=60.8$$

Question: If $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$ then

$\sin\left(x^2+x+5\right)\frac{\pi}{2}+\cos\left(x^2+x+5\right)\pi=?$

Answer: 0.00

Solution:

$$\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$$

$$\sin^{-1}\left(\frac{x+1}{\sqrt{(x+1)^2+1}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$$

$$\tan^{-1}(x+1)-\tan^{-1}x=\frac{\pi}{4}$$

Apply tan on both sides

$$\frac{(x+1)-x}{1+x^2+x}=1$$

$$\Rightarrow x^2+x=0$$

$$x=0,-1$$

Now, $\sin\left(x^2+x+5\right)\frac{\pi}{2}+\cos\left(x^2+x+5\right)\pi=$

Put $x=0$

$$\sin\frac{5\pi}{2}+\cos(5\pi)=1-1=0$$

Question: $g(x)=\sqrt{x}+1$ and $g(f(x))=x+3-\sqrt{x}$, then find $f(0)$.

Answer: 4.00

Solution:

Given, $g(x)=\sqrt{x}+1$ and $g(f(x))=x+3-\sqrt{x}$

$\sqrt{f(x)}+1=x+3-\sqrt{x}$

Put $x=0$

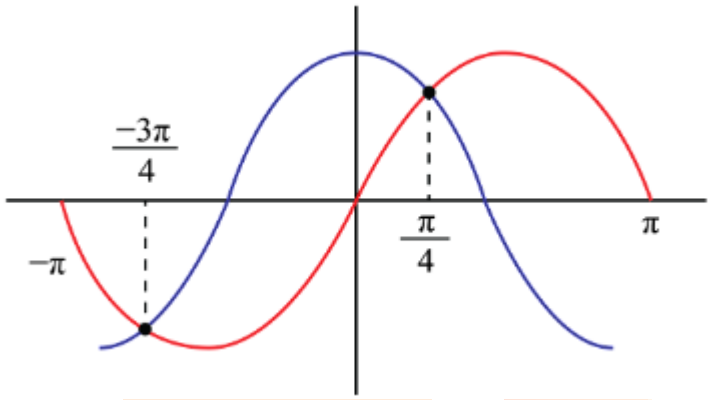
$\sqrt{f(0)}=2$

$f(0)=4$

Question: $y=\max(\sin x,\cos x),-\pi\le x\le\pi$. Find area.

Answer: 4.00

Solution:



$$\int_{-\pi}^{-\frac{3\pi}{4}}-\sin x+\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{2}}-\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}}\cos x+\int_{\frac{\pi}{4}}^{\pi}\sin x$$

$$\cos x\Big|_{-\pi}^{-3\pi/4}+(-\sin x)\Big|_{-3\pi/4}^{-\pi/2}+\sin x\Big|_{-\pi/2}^{\pi/4}+(-\cos x)\Big|_{\pi/4}^{\pi}$$

$$=\left(\frac{-1}{\sqrt{2}}+1\right)+\left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}+1\right)+\left(1+\frac{1}{\sqrt{2}}\right)$$

$=4$

Question: $S_1:\lim_{n\to\infty}\frac{1}{n^2}(2+4+6+\ldots+2n)=1$

$S_2:\lim_{n\to\infty}\frac{1}{n^{16}}(1^{15}+2^{15}+3^{15}+\ldots+n^{15})=\frac{1}{16}$

Options:

(a) Both are correct

(b) Both are incorrect

(c) Only S_1 is correct

(d) Only S_2 is correct

Answer: (a)

Solution:

$$S_1 : \lim_{n\to\infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow \text{ True}$$

$$S_2 : \lim_{n\to\infty} \frac{1}{n^{16}}\left(\sum r^{15}\right) = \lim_{n\to\infty} \frac{1}{n}\sum\left(\frac{r}{n}\right)^{15}$$

$$= \int_0^1 x^{15}dx = \frac{1}{16} \Rightarrow \text{ True}$$

Question: Find fractional part of $\frac{4^{2022}}{15}$.

Answer: $\frac{1}{15}$

Solution:

$$\left\{\frac{4^{2022}}{15}\right\} = \left\{\frac{2^{4044}}{15}\right\}$$

$$= \left\{\frac{(1+15)^{1011}}{15}\right\}$$

$$= \frac{1}{15}$$

Question: If $y_1(x)$ and $y_2(x)$ are solutions of $\frac{dy}{dx} = y + 7,\ y_1(0),\ y_2(0) = 1$ then find number of points of intersection of $y_1(x)$ and $y_2(x)$.

Answer: 0.00

Solution:

$$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$$

I.F. = e^{-x}

$$ye^{-x} = \int 7e^{-x}dx$$

$$\Rightarrow ye^{-x} = -7e^{-x} + c$$

$$\Rightarrow y = -7 + ce^{x}$$

$$y_1(x) = -7 + 7e^{x},\ y_2(x) = -7 + 8e^{x}$$

$$-7 + 7e^{x} = -7 + 8e^{x}$$

$$\Rightarrow e^{x} = 0$$

No solution

Question: $2\cdot 2^2 - 3^2 + 2\cdot 4^2 - 5^2 + \ldots$. Find sum till 20 terms.

Answer: 1310.00
Solution:

$$\left(2^3-3^2+4^2-5^2+\ldots 20 \text{ terms}\right)+\left(2^2+4^2+\ldots+10 \text{ terms}\right)$$

$$-\left(2+3+4+5+\ldots.+11\right)+4\left[1+2^2+\ldots.+10^2\right]$$

$$-\left[\frac{21\times 22}{2}-1\right]+4\times\frac{10\times 11\times 21}{6}$$

$$=1-231+14\times 11\times 10$$

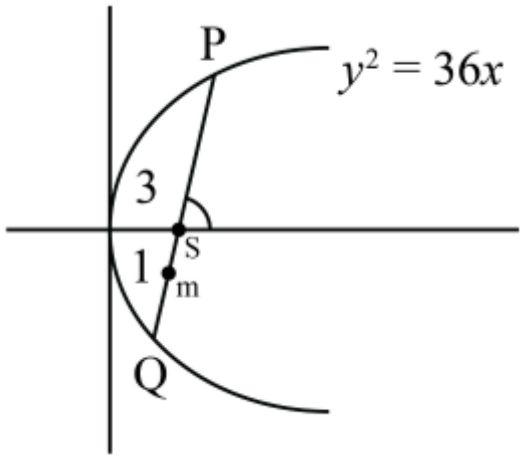
$$=1540+1-231$$

$$=1310$$

Question: Let PQ be a focal chord of the parabola $y^2=36x$ of length 100, making an acute angle with the positive and M be the point on the line segment PQ such that $PM:PQ=3:1$, then which of the following point does not lie on the line passing through M and perpendicular to the line PQ?

Answer:

Solution:



$$y^2=36x \Rightarrow a=9$$

$$A\equiv\left(9t^2,18t\right)$$

$$a\left(t+\frac{1}{t}\right)^2$$

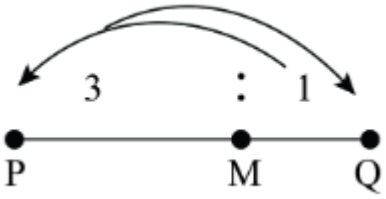
$$9\left(t+\frac{1}{t}\right)^2=100$$

$$t+\frac{1}{t}=\pm\frac{10}{3}$$

$$t=\pm 3 \;\&\; \pm\frac{1}{3}$$

$$Q\equiv\left(81,54\right)$$

$$P\equiv\left(1,-6\right)$$



$M=(61,39)$

Question: The negation of the statement $\left[(A\wedge(B\vee C))\Rightarrow(A\vee B)\right]\Rightarrow A$ is

Answer: $\sim A$

Solution:

$\left[(A\wedge(B\vee C))\Rightarrow(A\vee B)\right]\Rightarrow A$

$\left[\sim(A\wedge(B\vee C))\vee(A\vee B)\right]\Rightarrow A$

$\Rightarrow(A\wedge(B\vee C))\wedge(\sim A\wedge\sim B)\vee A$

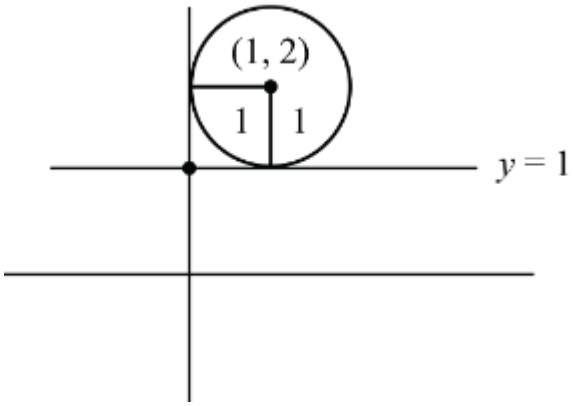
$\phi\vee A$

$=\sim A$

Question: $w=z(\bar{z})+k_1(z)+k_2(iz)+\lambda$. If $\mathrm{Re}(w)=0$ is a circle C in 1st quadrant with radius 1 touching line $y=1$ and y-axis. Then $\mathrm{Im}(w)=0$ intersects C at A & B, find $(AB)^2$.

Answer: 4.00

Solution:



$w=x^2+y^2+k_1(x+iy)+k_2(ix-y)+\lambda$

$\Rightarrow x^2+y^2+k_1x-k_2y+\lambda=0$

Centre $=\left(\frac{-k_1}{2},\frac{k_2}{2}\right)$

$r=\sqrt{\frac{k_1^2}{4}+\frac{k_2^2}{4}-\lambda}=1$

$\mathrm{Re}(w)=0$

$k_2^2=4\lambda$

$$\left|\frac{k_2}{2}-1\right|=1$$

$$\Rightarrow \frac{k_2}{2}-1=\pm 1$$

$$\Rightarrow \frac{k_2}{2}=2,0$$

$$\Rightarrow k_2=4,0$$

$$k_2=4,\ \lambda=4$$

$$\frac{k_1^{\,2}}{4}=1$$

$$\Rightarrow k_1=-4$$

Circle: $x^2+y^2-4x-4y+4=0$

$\mathrm{Im}(w)=0$

$k_1y+k_2x=0$

$x-y=0$

AB is diameter

$AB^2=4$

Question: In $\left(\sqrt{x}-\frac{6}{x^{\frac{3}{2}}}\right)^n$, constant term is A & sum of coefficient other than A = 649. If coefficient of x^{-n} is λA, then $\lambda=?$

Answer:

Solution:

$$T_{k+1}={}^nC_k\,(x)^{\frac{n-k}{2}}(-6)^k(x)^{\frac{-3}{2}k}$$

$$\frac{n-k}{2}-\frac{3}{2}k=0$$

$$n-4k=0$$

$$(-5)^n-\left({}^nC_{\frac{n}{4}}(-6)^{\frac{n}{4}}\right)=649$$

$n=4$

$625+24=649$

$n=4\ \&\ k=1$

Required is coefficient of x^{-4} is $\left(\sqrt{x}-\frac{6}{x^{\frac{3}{2}}}\right)^4$

${}^4C_1(-6)^3$

By calculating we will get $\lambda=36$

Question: Find maximum value: $f(x)=x-2\sin x\cos x+\frac{1}{3}\sin(3x),\ x\in[0,\pi]$

Answer: $\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

Solution:

$f(x)=x-2\sin x\cos x+\frac{1}{3}\sin(3x)$

$f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$

$f'(x)=1-2\cos 2x+\cos 3x=0$

$0=1-2(2\cos^2 x-1)+(4\cos^3 x-3\cos x)$

$0=1-4\cos^2 x+2+4\cos^3 x-3\cos x$

$4\cos^3 x-4\cos^2 x-3\cos x+3=0$

$4\cos^2 x(\cos x-1)-3(\cos x-1)=0$

$(\cos x-1)(4\cos^2 x-3)=0$

$\therefore x=0,\frac{5\pi}{6},\frac{\pi}{6}$

$f'(x)=1-2\cos 2x+\cos 3x=0$

$\therefore x=0,\frac{5\pi}{6},\frac{\pi}{6}$

$f''(x)=4\sin 2x-3\sin 3x$

$f''(0)=0$

$f''\left(\frac{\pi}{6}\right)>0$

$f''\left(\frac{5\pi}{6}\right)>0\Rightarrow\left(\frac{5\pi}{6}\right)$ is point of maxima

$\therefore\ f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

Question: The set of all $a\in R$ for which the equation $x|x-1|+|x+2|+a=0$ has exactly one real root is

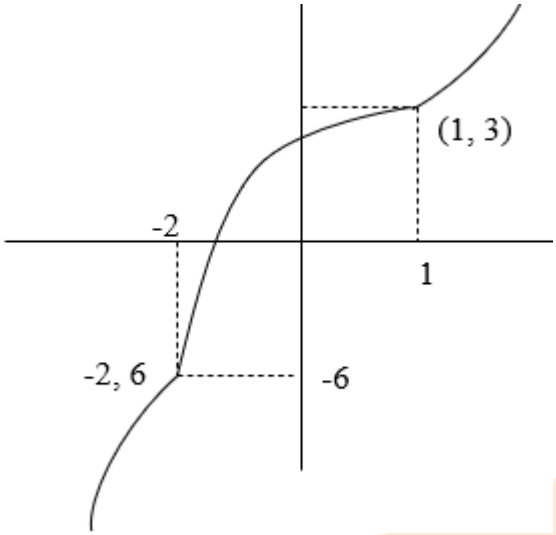
Answer: $a\in R$

Solution:

$f(x)=x|x-1|+|x+2|$

$x|x-1|+|x+2|+a=0$

$x|x-1|+|x+2|=-a$



All values are increasing.

Question: Let $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}, \alpha>2$ be the adjoint of a matrix A and $|A|=2$, then

$$\begin{bmatrix}\alpha & -2\alpha & \alpha\end{bmatrix} B \begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix} = ?$$

Answer: $[-16]_{1\times1}$

Solution:

Given, $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$

$|B|=4$

$1(8-3\alpha)-3(4-3\alpha)+\alpha(\alpha-2\alpha)=4$

$-\alpha^2+6\alpha-8=0$

$\alpha=2,4$

Given $\alpha>2$

So $\alpha=2$ is rejected

$$\begin{bmatrix}4 & -8 & 4\end{bmatrix}\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}4\\ -8\\ 4\end{bmatrix}=[-16]_{1\times1}$$