PART : MATHEMATICS

1. If α, β are real roots of $ax^2 + bx + 1 = 0$ then $\lim_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos(x^2 + bx + a)}{2(\alpha x - 1)^2} \right)^{\frac{1}{2}} = K\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$ then k is

(where $\alpha > \beta > 0$)

(1) $\frac{1}{\alpha}$ (2) $\frac{1}{2\alpha}$ (3) α (4) 2α

Ans. (2)

Sol. Equations whose roots and $\frac{1}{\alpha}, \frac{1}{\beta}$ is $a + bx + x^2 = \left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)$

$$\lim_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos\left(\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\right)}{2\alpha^2\left(x - \frac{1}{\alpha}\right)^2\left(x - \frac{1}{\beta}\right)^2} \right)^{\frac{1}{2}} \times \left|x - \frac{1}{\beta}\right| = \left(\frac{1}{2.2\alpha^2}\right)^{\frac{1}{2}} \left|\frac{1}{\alpha} - \frac{1}{\beta}\right| = \frac{1}{2\alpha}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

2. If f is a function defined from A → B such that A = {a, b, c, d, e} and B = {1, 2, 3, 4} then find number of onto function if f (a) = 1

Ans. (60)

Sol. Total no. of onto function

$= {}^4C_1 \times 3! + \frac{4!}{2!.2!.1!} \times \frac{1}{2!} \times 3!$

= 24 + 36

= 60

3. A be a set such that A= {1,2,3,4,5,6} and R be a relation on A such that x + y =7 then relation R is

(1) Reflexive (2) Symmetric (3) Transitive (4) Equivalence

Ans. (2)

Sol. Reflexive :- $xRx \Rightarrow x + x = 7$. $\Rightarrow x \notin A$. R is not reflexive.

Symmetric :- $xRy \Rightarrow x + y = 7$. $\Rightarrow y + x = 7 \Rightarrow yRx \Rightarrow$ R is symmetric

Transitive :- 1R6 & 6 R 1 but 1 is not related to 1 so not transitive

4. If $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(1) By 14 but not by 34 (2) By 34 but not by 14

(3) Neither by 34 or 14 (4) Both by 14 & 34

Ans. (2)

Sol. $(25^{190} - 19^{190}) - (8^{190} - 2^{190})$ is divisible by 6.

$(25^{190} - 8^{190}) - (19^{190} - 2^{190})$ is divisible by 17.

$25^{190} - 8^{190}$ is not divisible by 7

but $19^{190} - 2^{190}$ is divisible by 7

So, $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by 34 but not 14

5. Tangents OP and OQ are drawn from origin O to circle $x^2 + y^2 - 6x + 4y + 8 = 0$ if circumcircle of triangle OPQ passes through a point $P(\alpha, \frac{1}{2})$ then α can be.

(1) $\frac{5}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) 1

Ans. (1)

Sol. Equation of circumcircle whose diametric end point is (3, – 2) & (0,0)

$x(x - 3) + y(y + 2) = 0$

$x^2 + y^2 - 3x + 2y = 0$

pt $(\alpha, \frac{1}{2})$ is on circle

$\alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$

$\alpha^2 - 3\alpha + \frac{5}{4} = 0$

$4\alpha^2 - 12\alpha + 5 = 0$

$\alpha = \frac{10}{4}, \alpha = \frac{2}{4}$

$\alpha = \frac{5}{2}, \frac{1}{2}$

6. The absolute difference of coefficient of x^7 and x^{10} in $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is

(1) $12^3 - 12$ (2) $11^3 - 12$ (3) $13^3 - 13$ (4) $10^3 - 10$

Ans. (1)

Sol. $T_{r+1} = {}^{11}C_r (2x^2)^{11-r} (2x)^{-r}$

for coeff. x^7 put $22 - 2r - r = 7$

$r = 5$

and for coeff x^{10} put $22 - 3r = 10$

$r = 4$

abs . diff. $= \left| {}^{11}C_5 \cdot \frac{2^6}{2^5} - {}^{11}C_4 \cdot \frac{2^7}{2^4} \right|$

$|924 - 2640| = 1716 = 12^3 - 12$

7. $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x \le 1 \\ mx^2 + k^2, & x > 1 \end{cases}$ (where $k \ne 0$)

f(x) is differential for x > 0, then $\dfrac{8.f'(8)}{f'\left(\frac{1}{8}\right)}$ = ...

Ans. (309)

Sol. $f(1^-) = f(1) = f(1^+)$

$3 + k\sqrt{2} = m + k^2$ ---------(1)

$f'(1^-) = f'(1^+)$

$6 + \dfrac{k}{2\sqrt{2}} = 2m$ ----------(2)

from (1) & (2)

$3 + k\sqrt{2} = 3 + \dfrac{k}{4\sqrt{2}} + k^2$

$k^2 + k\left(\dfrac{1}{4\sqrt{2}} - \sqrt{2}\right) = 0$

$k = 0, k = \dfrac{7}{4\sqrt{2}}$

If $k = \dfrac{7}{4\sqrt{2}}$, $m = 3 + \dfrac{7}{32} = \dfrac{103}{32}$

$f'(x) = \begin{cases} 6x + \dfrac{k}{2\sqrt{x+1}} \\ 2mx \end{cases}$

$f'(8) = \dfrac{103}{2}$

$f'\left(\dfrac{1}{8}\right) = \dfrac{6}{8} + \dfrac{k.2\sqrt{2}}{2.3} = \dfrac{3}{4} + \dfrac{\sqrt{2}k}{3} = \dfrac{4}{3}$ now $\dfrac{8.f'(8)}{f'\left(\frac{1}{8}\right)} = 309$

8. If $x > y > z > 0$, $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ are in A.P., $x, \sqrt{2}y, z$ are in G.P. and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{3}{\sqrt{2}}$ then $3(x + y + z)^2$

Ans. (150)

Sol. $\dfrac{2}{y} = \dfrac{1}{x} + \dfrac{1}{z}$ ......(1)

$\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{3}{\sqrt{2}} \Rightarrow \dfrac{3}{y} = \dfrac{3}{\sqrt{2}}$

$y = \sqrt{2}$ ......(2)

$x, \sqrt{2}y, z$ are in G. P.

$2y^2 = xz$

$xz = 4$.........(3)

from (1) $x + z = \sqrt{2}\, xz$

$x + z = 4\sqrt{2}$

now $3(x + y + z)^2 = 3(4\sqrt{2} + \sqrt{2})^2 = 150$

9. Let $A = \begin{bmatrix} 1 & \lambda \\ 5 & 10 \end{bmatrix}$ for which $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = 2$ the value of $\alpha^2 + \beta^2 + \lambda^2 =$

(1) $\frac{346}{25}$ (2) $\frac{147}{25}$ (3) $\frac{11}{25}$ (4) $\frac{354}{25}$

Ans. (2)

Sol. $|A - xI| = 0$ $|A| \neq 0 \Rightarrow \lambda \neq 2$

$\begin{bmatrix} 1-x & \lambda \\ 5 & 10-x \end{bmatrix} = 0$

$x^2 - 11x + (10 - 5\lambda) = 0$

$\therefore A^2 - 11A + (10 - 5\lambda) I = 0$

$\Rightarrow A - 11 I + (10 - 5\lambda) A^{-1} = 0$

$A^{-1} = \frac{1}{(5\lambda - 10)} A + \frac{-11}{(5\lambda - 10)} I$

$\alpha = \frac{1}{5\lambda - 10}$, $\beta = \frac{-11}{5\lambda - 10}$

$\alpha + \beta = 2 \Rightarrow \frac{-10}{5\lambda - 10} = 2$

$\lambda = 1$

$\alpha = -\frac{1}{5}$ $\beta = \frac{11}{5}$

$\alpha^2 + \beta^2 + \lambda^2 = \frac{1}{25} + \frac{121}{25} + 1 = \frac{147}{25}$

10. Probability distribution of variable 'x' is given by $P((X = x) = K(1 + x) \cdot 3^{-x}$, $x = 0, 1, 2, 3, \ldots\ldots, \infty$ then $P(x \geq 2) =$

(1) 28 (2) $\frac{7}{27}$ (3) $\frac{9}{28}$ (4) $\frac{11}{27}$

Ans. (2)

Sol. $P(x = 0) + P(x = 1) + p(x = 2) + p(x = 3) + \ldots\ldots = 1$

$\frac{K}{3^0} + \frac{2K}{3^1} + \frac{3K}{3^2} + \frac{4K}{3^3} + \ldots\ldots\infty = 1$

$K\left(1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots\ldots\infty\right) = 1$

Now

Let $S = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots\infty$

$\frac{S}{3} = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots\infty$

$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\infty$

$= \frac{1}{1 - \frac{1}{3}}$

$S = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

Now $K.\frac{9}{4} = 1$

$K = \frac{4}{9}$

Now $P(x \geq 2) = P(2) + P(3) + \ldots\ldots \infty$

$= 1 - P(0) - P(1)$

$= 1 - \left(\frac{K}{1} + \frac{2K}{3}\right) = 1 - \frac{5K}{3}$

$1 - \frac{20}{27} = \frac{7}{27}$

11. $\int_0^{2.4} [x^2]dx$

(1) $9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$ (2) $9 + \sqrt{2} - \sqrt{3} - \sqrt{5}$ (3) $10 - 2 - \sqrt{3} - \sqrt{5}$ (4) $9 + \sqrt{2} + \sqrt{3} + \sqrt{5}$

Ans. (1)

Sol. $= \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx + \int_2^{\sqrt{5}} 4.dx + x\int_{\sqrt{5}}^{2.4} 5.dx$

$= (x)_1^{\sqrt{2}} + 2(x)_{\sqrt{2}}^{\sqrt{3}} + 3(x)_{\sqrt{3}}^{2} + 4(x)_2^{\sqrt{5}} + 5(x)_{\sqrt{5}}^{2.4}$

$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) + 4(\sqrt{5} - 2) + 5(2.4 - \sqrt{5})$

$= -1 - \sqrt{2} - \sqrt{3} - 2 + 12 - \sqrt{5}$

$= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$

12. The mean and variance of 12 observations are $\frac{9}{2}$ and 4 respectively. Two observations 7 & 14 are misread as 9 & 10. The correct valance is m/n then m + n is equal to

(1) 314 (2) 315 (3) 316 (4) 317

Ans. (317)

Sol. $\frac{\sum x_i}{12} = \frac{9}{2} \Rightarrow \sum x_i = 54.$

correct $\sum x_i = 54 + 7 + 14 - 9 - 10 = 56.$

correct $\bar{x}_i = \frac{56}{12} = \frac{14}{3}$

$\sigma^2 = \frac{\sum x_i^2}{12} - \left(\frac{9}{2}\right)^2 = 4$

$\Rightarrow \frac{\sum x^2}{12} = \frac{97}{4} \Rightarrow \sum x_i^2 = 291$

corr. $\sum x_i^2 = 291 + 64 = 355$

corr $\sigma^2 = \frac{\sum x^2}{12} - (\bar{x})^2$

$= \frac{355}{12} - \left(\frac{14}{3}\right)^2$

$= \frac{281}{36} = \frac{m}{n}$

$m + n = 281 + 36 = 317$

13. $\vec{V}_1 = a\hat{i} + \hat{j} + \hat{k}$

$\vec{V}_2 = \hat{i} + b\hat{j} + \hat{k}$

$\vec{V}_3 = \hat{i} + \hat{j} + c\hat{k}$

are coplanar and vector

$\vec{u}_1 = (a + b)\hat{i} + c\hat{j} + c\hat{k}$

$\vec{u}_2 = a\hat{i} + (b + c)\hat{j} + a\hat{k}$

$\vec{u}_3 = b\hat{i} + b\hat{j} + (a + c)\hat{k}$

are also coplanar then the value of 6 (a + b + c)

Ans. **12**

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$abc + 2 - a - b - c = 0$

& $\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & a+c \end{vmatrix} = 0$

$R_3 \to R_3 - (R_1 + R_2)$

$\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0$

$-2ac^2 - 2a^2c - 2ac(b + c) - 2ac(a + b) = 0$

$ac^2 + a^2c - abc - ac^2 - a^2c - abc = 0$

$-2abc = 0$

$abc = 0$

$2 - a - b - c = 0$

$a + b + c = 12$ $\qquad$ $6(a + b + c) = 12$

14. Negation of $(p \wedge \sim q) \vee (\sim p)$ is.

(1) $p \wedge q$ (2) $\sim (p \wedge q)$ (3) $p \wedge \sim q$ (4) $p \vee q$

Ans. (1)

Sol. $\sim [(p \wedge \sim q) \vee (\sim p)]$

$\sim [(p \wedge \sim q) \wedge (p)]$

$(\sim p \vee q) \wedge (p) = (\sim p \wedge p) \vee (q \wedge p)$

$C \vee (p \wedge q) = (p \wedge q)$

15. The number of words with or without meaning can be formed form the word "MATHEMATICS" where C and S are not together is k × 6! then k is

(1) 5670 (2) 818 (3) 1636 (4) 409

Ans. (1)

Sol. Required no. of ways = $\frac{11!}{2!2!2!} - \frac{10!}{2!2!2!} \times 2!$

$= \frac{9 \times 10!}{8}$

now $\frac{9 \times 10!}{8} = k \times 6!$

$k = 9 \times 9 \times 10 \times 7$

$= 5670$

16. Find area included by lines $x + y = 2$, $x = 0$, $y = 0$ and $f(x) = \min.\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$, where [.] is greatest integral part.

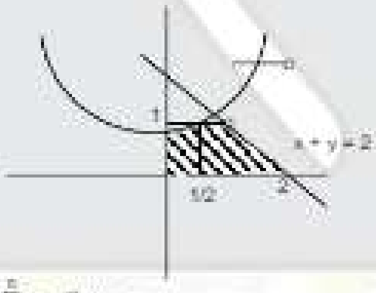
(1) $\frac{17}{12}$ (2) $\frac{19}{12}$ (3) $\frac{23}{12}$ (4) $\frac{25}{12}$

Ans. (1)

Sol. $A = \int_0^{\frac{1}{2}} \left(x^2 + \frac{3}{4}\right) dx + \frac{1}{2}\left(\frac{3}{2} + \frac{1}{2}\right) \cdot 2$

$= \frac{1}{3} \cdot \frac{1}{8} + \frac{3}{4} \cdot \frac{1}{2} + 1$

$12A = \frac{1}{2} + \frac{9}{2} + 12 = 17$



17. Let $a_n = 5 + 8 + 14 + 23 + \ldots$ upto n terms. If $S_n = \sum_{k=1}^{n} a_k$, then $S_{30} - a_{40}$ is equal to

(1) 78025 (2) 11290 (3) 12390 (4) 13490

Ans. (2)

Sol. $T_n = An^2 + Bn + C$

$T_1 = A + B + C = 5$

$T_2 = 4A + 2B + C = 8$

$T_3 = 9A + 3B + C = 14$

$\Rightarrow A = B = 3 - \frac{9}{2} = \frac{-3}{2}$

$C = 5$

$a_n = T_n = \frac{3n^2}{2} - \frac{3n}{2} + 5$

$a_{40} = \frac{3 \times 40^2}{2} - \frac{3 \times 40}{2} + 5$

$= 2400 - 60 + 5$

$= 2345$

$S_{30} = \frac{3}{2}\sum n^2 - \frac{3}{2}\sum n + \sum 5$

$= \frac{3}{2} \times \frac{30 \times 31 \times 61}{6} - \frac{3}{2} \times \frac{30 \times 31}{2} + 150$

$= \frac{3}{2} \times 5 \times 1891 - \frac{1395}{2} + 150$

$= \frac{25365 - 1395 + 300}{2}$

$= 13635$

So $S_{30} - a_{40} = 13635 - 2345$

$= 11290$