JEE-Mains-10-04-2023 [Memory Based] [Evening Shift]

Physics

Question: Two projectiles were projected at angles of 30° and 60° with the same velocity. Find the ratio of their maximum heights

Options:

(a) 1 : 1

(b) 1 : 2

(c) 1 : 3

(d) 1 : 4

Answer: (c)

Solution:

$$\frac{H_1}{H_2}=\frac{\frac{u^2\sin^2 30}{2g}}{\frac{u^2\sin^2(60)}{2g}}=\frac{\sin^2 30}{\sin^2 60}=\frac{1}{3}$$

Question: Two wires of same length with area of cross sections in the ratio 1:3 have the same load applied. Their Young's moduli are in the ratio 1 : 4. Find the ratio of elongations

Options:

(a) 12 : 1

(b) 20 : 1

(c) 5 : 1

(d) 1 : 5

Answer: (a)

Solution:

$$\Delta L=\frac{FL}{AY}$$

$$\frac{\Delta L_1}{\Delta L_2}=\frac{\frac{FL}{A_1Y_1}}{\frac{FL}{A_2Y_2}}=\frac{A_2Y_2}{A_1Y_1}=\frac{3\times 4}{1\times 1}=\frac{12}{1}$$

Question: S1: Acceleration due to gravity is affected by latitude

S2: Maximum at poles & minimum at equator

Options:

(a) Both are true

(b) S1 is true, S2 false

(c) S2 is true, S1 false

(d) Both are false

Answer: (a)

Solution:

Question: On a straight road a car moves half distance with velocity v_1 and remaining half with velocity v_2 then the average velocity is

Options:

(a) $2v_1v_2/v_1 + v_2$

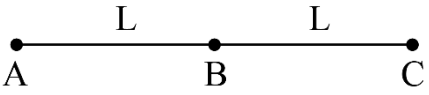
(b) $v_1v_2/v_1 + v_2$

(c) $v_1 + v_2/2v_1v_2$

(d) None of this

Answer: (a)

Solution:

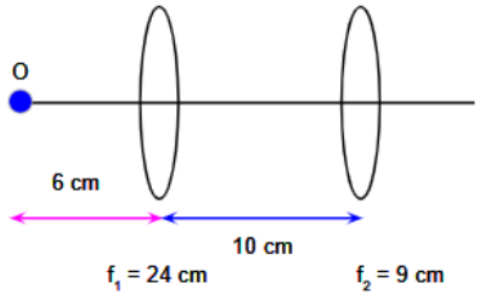


$$v_1 = \frac{L}{t_1}; r_2 = \frac{L}{t_2}$$

$$t_1 = \frac{L}{v_1}; t_2 = \frac{L}{v_2}$$

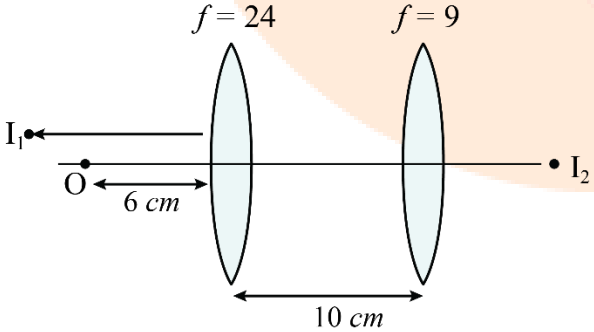
$$v_{avg} = \frac{2L}{t_1 + t_2} = \frac{2L}{\frac{L}{v_1} + \frac{L}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

Question: Find the distance between object and image



Answer: 34.00

Solution:



For $I_1 \Rightarrow \frac{1}{24} = \frac{1}{V} - \frac{1}{(-6)} \Rightarrow \frac{1}{V} = \frac{1}{-8}$

I_1 is behind object

For $I_2 \Rightarrow$

$$\frac{1}{g} = \frac{1}{V} - \frac{1}{(-18)} \Rightarrow \frac{1}{V}$$

So V = +18

So distance between object and image is 6 + 10 + 18 = 34

Question: Find v_2 if $A_1 = 2\ cm^2$, $A_2 = 10\ mm^2$, $v_1 = 4$ cm/s
Answer: 80.00
Solution:

$$A_1V_1 = A_2V_2$$

$$2\,\text{cm}^2 \times 4\frac{\text{cm}}{\text{s}} = 10\,\text{mm}^2 \times V_2$$

$$2\,\text{cm}^2 \times 4\frac{\text{cm}}{\text{s}} = 10 \times 10^{-2}\,\text{cm}^2 \times V_2$$

$$V_2 = 80\ \text{cm/s}$$

Question: Find amplitude of SHM y = sin(ωt) + cos(ωt)
Options:
(a) $\sqrt{3}$
(b) $\sqrt{2}$
(c) $\sqrt{5}$
(d) 2
Answer: (b)
Solution:

$$y = \sin(\omega t) + \cos(\omega t)$$

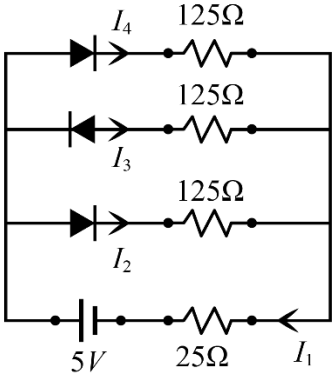
$$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin(\omega t) + \frac{1}{\sqrt{2}}\cos(\omega t)\right)$$

$$\sqrt{2}\sin\left(\omega t + 45°\right)$$

$$A = \sqrt{2}$$

Question: An object moves x distance with speed v_1 and next x distance with speed v_2. The average velocity v is related to v_1 and v_2 as
Options:
(a) $v = \frac{(v_1 + v_2)}{2}$
(b) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$
(c) $v = \left(\frac{2v_1v_2}{v_1 + v_2}\right)$
(d) $v = \left(\frac{v_1 - v_2}{2}\right)$
Answer: (c)
Solution: $v = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$

Question: Following circuit contains diodes with forward bias having resistance 25 Ω and reverse bias having infinite resistance. The ratio of I_1/I_2 is equal to



Options:

(a) 1

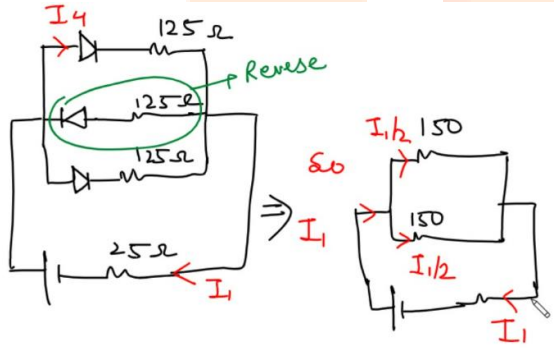
(b) 2

(c) 3

(d) 4

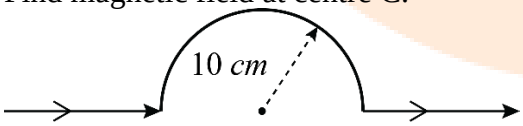
Answer: (b)

Solution:



$$I_4 = \frac{I_1}{2} \text{ so } \frac{I_1}{I_4} = 2$$

Question: An infinitely-long conductor has a current 14 A flowing as shown in the figure. Find magnetic field at centre C.



Options:

(a) 88μT

(b) 44μT

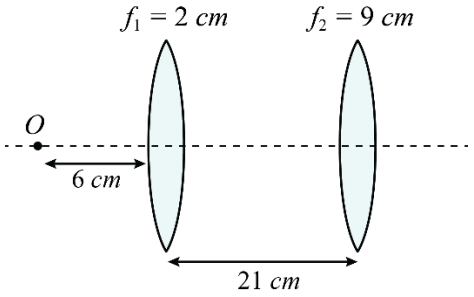
(c) 10μT

(d) 120μT

Answer: (b)

Solution: $\frac{\mu_0 I}{4r}$

Question: A point object (0) is placed on the principle axis of a system of two lenses as shown. Find the distance between the image and the object.



Options:
(a) 45 cm
(b) 40 cm
(c) 55 cm
(d) 50 cm
Answer: (a)
Solution:

$$\frac{1}{v}-\frac{T}{-6}=\frac{1}{2}$$

$$\frac{1}{v}=\frac{3+1}{3+2}-\frac{1}{6}=\frac{2}{6}$$

$$\frac{1}{v}=\frac{1}{-18}=\frac{1}{9}$$

$$\frac{1}{v}=\frac{1}{9}-\frac{1}{18}=\frac{1}{18}=18$$

Question: If half life for a radio active decay reaction is T. Find the time after which $\frac{7}{8}^{th}$ of initial mass decays
Options:
(a) T
(b) 2T
(c) 3T
(d) 4T
Answer: (c)
Solution:

$$N=\frac{N_0}{2^{\left(\frac{t}{T}\right)}}$$

So,

$$\frac{N_0}{8}=\frac{N_0}{2^{(t/T)}}\Rightarrow 2^3=2^{t/T}$$

$$t=3T$$

Question: If min wavelength of layman series is 917 Å find min wavelength of balmer series
Options:
(a) 1992 Å
(b) 4000 Å
(c) 2668 Å
(d) 3668 Å

Answer: (d)
Solution:

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\text{so } \frac{\frac{1}{(\lambda_{\text{lyman}})_{\min}}}{\frac{1}{(\lambda_{\text{Balmer}})_{\min}}} = \frac{RZ^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]}{RZ^2\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]}$$

$$\frac{\lambda_B}{\lambda_L} = \frac{1}{1} \times \frac{4}{1} \Rightarrow \lambda_B = 4\lambda_L = 4 \times 917 = 3668$$

Question: An air capacitor has capacitor C_1 when metal sheet of thickness 2D/3 is inserted between plates the new capacity becomes C_2 find C_2:C_1
Answer: 3.00
Solution:

$$C_1 = \frac{\varepsilon_0 A}{d}$$

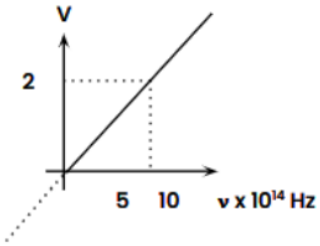
$$C_2 = \frac{\varepsilon_0 A}{\frac{2d}{3(\infty)} + \frac{d}{3(1)}} \Rightarrow \frac{3\varepsilon_0 A}{d}$$

$$\text{So } \frac{C_1}{C_2} = \frac{1}{3} \text{ or } \frac{C_2}{C_1} = \frac{3}{1}$$

Question: Assertion(A): fan spins even after switch is off
Reason (R): Fan in rotation has rotational inertia.
Options:
(a) A is correct and R is correct explanation of A
(b) A is correct and R is incorrect explanation of A
(c) A is correct and R is correct but R is not correct explanation of A
(d) Both (A) and (R) are incorrect
Answer: (a)

Question: When electric field is applied to the electrons in a conductor it starts
Options:
(a) Moving in straight line
(b) Drifting from higher potential to lower potential
(c) Drifting from lower potential to higher potential
(d) Moving with constant velocity
Answer: (c)

Question: Based on given graph between stopping potential and frequency of irradiation, work function of metal is equal to



Options:
(a) 1 eV
(b) 3 eV
(c) 2 eV
(d) 4 eV
Answer: (b)
Solution:

Question: S1 : Magnetic susceptibility of diamagnetic substance is $-1 \leq \chi < 0$.
S2 : Diamagnetic substance move from stronger to weaker magnetic field
Options:
(a) Both are true
(b) S1 is true, S2 false
(c) S2 is true, S1 false
(d) Both are false
Answer: (a)

Question: Frictional force acting on lift of mass 1400 kg is 2000N. If lift moves with constant velocity of 3 m/s in upward direction, the power (in kW) of motor is ($g = 10\ m/s^2$)
Answer: 48.00
Solution:

$$P = F_N \cdot v$$
$$= (mg + f) \cdot V$$
$$= (1400 \times 10 + 2000) \cdot 3$$
$$= 48000 \text{ or } 48\text{kW}$$

Question: In YDSE for identical sources resultant intensity is I_1 if phase diff. is $\pi/2$ and resultant intensity is I_2 if phase diff is $\pi/3$ find I_1/I_2
Options:
(a) 2 : 3
(b) 3 : 2
(c) 1 : 4
(d) 4 : 1
Answer: (a)
Solution:

$$\frac{I_1}{I_2} = \frac{I + I + 2I\cos(\pi/2)}{I + I + 2I\cos(\pi/3)}$$
$$\frac{I_1}{I_2} = \frac{2I}{3I} = \frac{2}{3}$$

Question: A satellite is revolving around Earth at height R from surface find its time period of motion ($g = \pi^2$)

Options:

(a) $\sqrt{32R}$

(b) $\sqrt{16R}$

(c) $\sqrt{(8R)}$

(d) $\sqrt{(4R)}$

Answer: (a)

Solution:

$$m_S \omega^2 (2R) = \frac{GM_E m_S}{(2R)^2}$$

$$\frac{4\pi^2}{T^2} 02R = \frac{GM_E}{4R^2}$$

$$\frac{\pi^2 R}{T^2} = \frac{g}{32}$$

$$\Rightarrow T^2 = 32R$$

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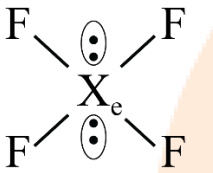
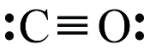
Chemistry

Question: Number of molecules having two lone pairs
F_2, XeF_4, NO, NO_2, CO, H_2O, CO_2
Options:
(a) 2
(b) 3
(c) 4
(d) 5
Answer: (b)
Solution:





Question: Which doesn't disturb balance of CO_2 and O_2?
Options:
(a) Burning of coal
(b) Respiration
(c) Burning of petroleum
(d) Photosynthesis
Answer: (d)
Solution: Green plants require CO_2 for photosynthesis and they, in turn, emit oxygen, thus maintaining the delicate balance.

Question: Correct order of Metallic character of the following elements
K, Be, Ca
Options:
(a) K > Be > Ca
(b) Ca > Be > K
(c) Be > Ca > K

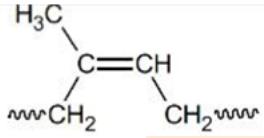
(d) K > Ca > Be
Answer: (d)
Solution: K > Ca > Be

Question: Why is $FeCl_3$ added to help clotting of blood?
Options:
(a) $FeCl_3$ reacts with blood
(b) Cl^- coagulates blood
(c) Fe^{3+} coagulates blood which is a negative sol
(d) Nine of the above
Answer: (c)
Solution: Fe^{+3} and blood contain negatively charged

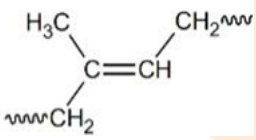
Hence, it stop bleeding due to coagulation.

Question: Which of the following is the correct structure of Buna S?
Options:
(a)

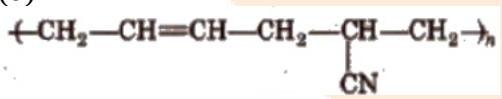
(b)



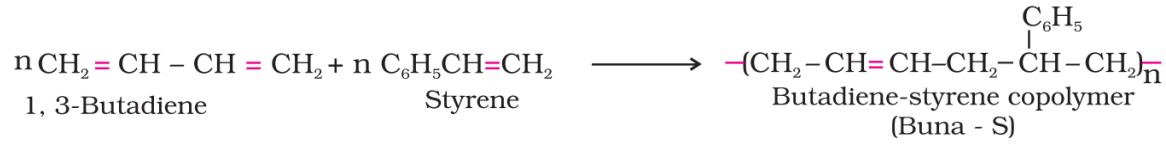
(c)



(d)



Answer: (a)
Solution:



Question: Which of the following is correct relation b/w radius (r) and edge length (a) for BCC and FCC respectively?
Options:
(a) $\frac{\sqrt{3}}{4}a, \frac{a}{2\sqrt{2}}$

(b) $\frac{a}{2\sqrt{2}}, \frac{\sqrt{3}}{4}a$

(c) $\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}$

(d) $\frac{\sqrt{3}}{4}a, \frac{\sqrt{3}}{4}a$

Answer: (a)

Solution:

$$\text{BCC } r = \frac{\sqrt{3}}{4}a$$

$$\text{FCC } r = \frac{a}{2\sqrt{2}}$$

Question: Number of unpaired electrons in ion, if its magnetic moment is 4.9 BM.

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (d)

Solution: The unpaired electron is 4.

If the spin magnetic moment is 4.9

Question: The difference of oxidation number of Xe in XeF_4 and XeO_3, when XeF_4 undergoes complete hydrolysis?

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (b)

Solution:

$$\underset{\text{Oxidation No} = 4}{6XeF_4} + 12H_2O \rightarrow 4Xe + \underset{\text{Oxidation No} = 6}{2XeO_3} + 24HF + 3O_2$$

Difference in oxidation state = 6 – 4 = 2

Question: Correct order of acidity of

A. Phenol

B. p-nitrophenol

C. p-methoxyphenol

D. tert-butyl alcohol

E. methanol

Options:

(a) B > D > C > E > A

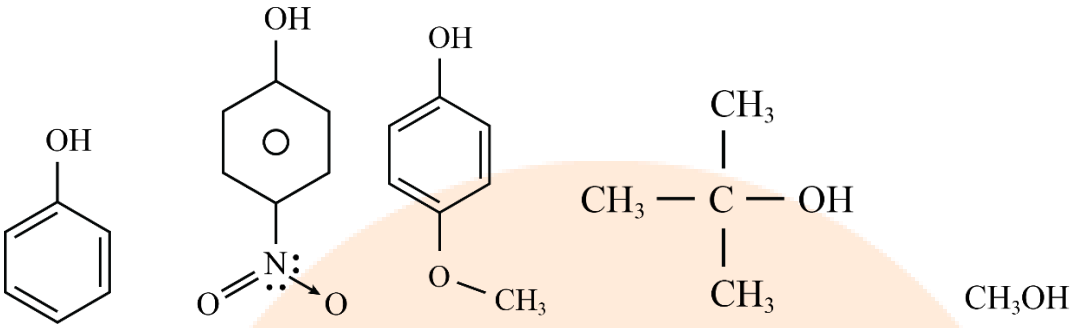
(b) E > C > D > B > A

(c) B > A > C > E > D

(d) E > A > D > C > B

Answer: (c)

Solution:



A B C D E

B > A > C > E > D

Question: Assertion: Mg^{2+} is more stable than Mg^{+}

Reason: Mg^{2+} is smaller and has more charge

Options:

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

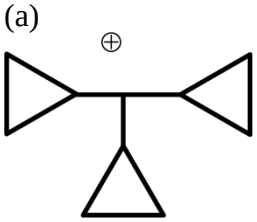
Answer: (b)

Solution: Assertion is correct and reason is also correct, but reason is not correct explanation of assertion.

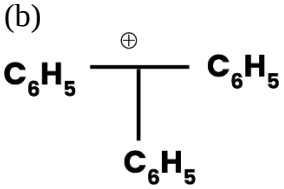
Question: The stability of carbocation is

Options:

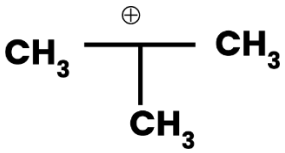
(a)



(b)



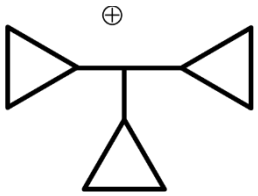
(c)



(d) $C_6H_5CH^{2+}$

Answer: (a)

Solution:



Question: Which of the following cannot be used to prepare alcohol?

Options:

(a) Ozonolysis

(b) Hydroboration Reaction

(c) Oxymercuration and Demercuration reaction

(d) Addition of dilute H_2SO_4 on alkene

Answer: (a)

Solution: Ozonolysis

Question: Number of electron taken by per manganate ion in alkaline medium

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (a)

Solution: Parmagnate ion is basic medium

$KMnO_4 + OH^{\ominus} \rightarrow MnO_4^{-2}$

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Mathematics

Question: Given that $g(x)=f(x)+f(1-x)$ and $f''(x)>0$ for $x\in(0,1)$. If $g(x)$ decreases in $x\in(0,\alpha)$ and increases in $x\in(\alpha,1)$, then

$$\tan^{-1}(\alpha)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)+\tan^{-1}(2\alpha)=?$$

Answer: π

Solution:

$$g(x)=f(x)+f(1-x)$$

$$\Rightarrow g'(x)=f'(x)-f'(1-x)$$

Put $x=\frac{1}{2}$

$$g'\left(\frac{1}{2}\right)=f'\left(\frac{1}{2}\right)-f'\left(1-\frac{1}{2}\right)$$

$$g'\left(\frac{1}{2}\right)=0$$

$$\Rightarrow g''(x)=f''(x)+f''(1-x)>0$$

$\Rightarrow$ Concave up

$$\Rightarrow \alpha=\frac{1}{2}$$

$$\Rightarrow \tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)$$

$$=\frac{\pi}{4}+\pi+\tan^{-1}\left(\frac{2+3}{1-(2)(3)}\right)$$

$$=\frac{\pi}{4}+\pi-\frac{\pi}{4}$$

$$=\pi$$

Question: $S_n = 4+11+21+34+....n$ terms. Find the value of $\frac{1}{60}\times\left[S_{29}-S_9\right]$.

Answer: 223.00

Solution:

$$S_n = 4+11+21+34+...+a_n$$
$$\underline{S_n = \quad 4+11+21+.....+....+a_n}$$
$$0 = \left[4+7+10+13+....+...\right]-a_n$$

$$a_n = \frac{n}{2}\left[2(4)+(n-1)3\right]$$
$$= \frac{n}{2}\left[3n+5\right]$$
$$= \frac{3n^2+5n}{2}$$

$$\Rightarrow S_{29} = \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$$
$$= \frac{3}{2}\cdot\frac{29(30)(59)}{6}+\frac{5}{2}\cdot\frac{29(30)}{2}$$
$$= \frac{29(10)}{4}\left[59+5\right]$$
$$= \frac{29(30)}{4}\left[64\right]$$
$$= 29(30)(16)$$

$$\Rightarrow S_9 = \frac{3}{2}\cdot\frac{9(10)(19)}{6}+\frac{5}{2}\cdot\frac{9(10)}{2}$$
$$= \frac{9(10)}{4}\left[19+5\right]$$
$$= \frac{9(10)}{4}(24)$$
$$= 9(10)(6)$$

$$\Rightarrow S_{29}-S_9 = 29(30)(16)-9(10)(6)$$
$$= (30)(2)\left[29(8)-9\right]$$
$$= 60(232-9)$$
$$= 60(223)$$

$$\therefore \quad \frac{1}{60}\times\left[S_{29}-S_9\right] = \frac{1}{60}\times 60(223)$$
$$= 223$$

Question: $\frac{x^2}{19}+\frac{x^2}{15}=1$ and $x^2+y^2=16$. Find angle between common tangent with minor axis of ellipse.

Answer: $m=\frac{1}{\sqrt{3}}$

Solution:

$\frac{x^2}{19}+\frac{x^2}{15}=1$

$y=mx\pm\sqrt{19m^2+15}$

$\Rightarrow x^2+y^2=16$

$y=mx\pm4\sqrt{1+m^2}$

$\Rightarrow \pm\sqrt{19m^2+15}=\pm4\sqrt{1-m^2}$

$19m^2+15=16\left(1+m^2\right)$

$19m^2+15=16+16m^2$

$19m^2-16m^2=16-15$

$3m^2=1$

$m^2=\frac{1}{3}\Rightarrow m=\pm\frac{1}{\sqrt{3}}$

$\Rightarrow \theta=30^\circ$ or $150^\circ\left(90^\circ+60^\circ\right)$

$\Rightarrow$ Angle with minor axis $=90^\circ-30^\circ=60^\circ$

Question: Q and P are orthocentre and circumcentre of ΔABC. $\overline{PA}+\overline{PB}+\overline{PC}$ in terms of $\overline{PQ}=?$

Options:

(a) $\overline{PQ}$

(b) $\overline{QP}$

(c) $2\overline{PQ}$

(d) $2\overline{QP}$

Answer: (a)

Solution:

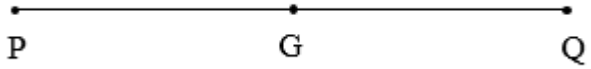
Let circumcentre P as origin.

$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$

$=\vec{a}+\vec{b}+\vec{c}$

$=3\left(\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

$=3\overrightarrow{PG}$



$\overrightarrow{PG}=\dfrac{\overrightarrow{PQ}}{3}$

$3\overrightarrow{PG}=\overrightarrow{PQ}$

Question: Find the sum of 4-digit numbers that can be formed using the digits 1, 3, 2, 2.
Answer: 26664.00
Solution:

The sum of 4-digit numbers is

$=\left[\dfrac{3!}{2!}(1+3)+3!(2)\right](1+10+100+1000)$

$=(12+12)(1111)$

$=(24)(1111)$

$=26664$

Question: In the expansion of $(x+1)^p(1-x)^q$, coefficient of x is 4 and coefficient of x^2 is -5. Find $2p+3q$.
Answer: 63.00
Solution:

$(x+1)^p(1-x)^q$

Coefficient of $x=-{}^pC_0\cdot{}^qC_1+{}^pC_1\cdot{}^qC_0$

$4=-q+p$

Coefficient of $x^2={}^pC_0\,{}^qC_2+{}^pC_2\,{}^qC_0-{}^pC_1\,{}^qC_1$

$$-5=\frac{q(q-1)}{2}+\frac{p(p-1)}{2}-pq$$
$$-10=q^2-q+p^2-p-2pq$$
$$-10=\left(p^2+q^2-2pq\right)-(p+q)$$

$$-10=(p-q)^2-(p+q)$$
$$-10=4^2-(p+q)$$
$$p+q=16+10=26$$

$$\Rightarrow 2p=26+4$$
$$\Rightarrow p=15$$
$$\Rightarrow q=11$$

$$2p+3q=2(15)+3(11)$$
$$=30+33$$
$$=63$$

Question: Let the lines through (1, 2) cuts the circle $x^2+y^2=16$ at A & B. Find locus of mid point of AB.
Answer: $x^2+y^2-x-2y=0$
Solution:

Let midpoint be (h,k)

Midpoint form of chord: $T=S_1$

$$hx+ky-16=h^2+k^2-16$$
$$hx+ky=h^2+k^2$$

Put point (1, 2)

$$h+2k=h^2+k^2$$
$$\Rightarrow x+2y=x^2+y^2$$
$$x^2+y^2-x-2y=0$$

Question: Consider $f(x)=\sec^{-1}\left(\frac{2x}{5x+3}\right)$. Domain of f is $[\alpha,\beta]\cup[\gamma,\delta]$. Then

$|3\alpha+10\beta+5\gamma+21\delta|=?$
Answer: 21.00
Solution:

$$\left|\frac{2x}{5x+3}\right| \geq 1$$

$$\frac{2x}{5x+3} \geq 1 \text{ or } \frac{2x}{5x+3} \leq -1$$

$$\frac{2x}{5x+3} - 1 \geq 0 \text{ or } \frac{2x}{5x+3} + 1 \leq 0$$

$$\frac{2x-5x-3}{5x+3} \geq 0 \text{ or } \frac{2x+5x+3}{5x+3} \leq 0$$

$$\frac{-3x-3}{5x+3} \geq 0 \text{ or } \frac{7x+3}{5x+3} \leq 0$$

$$\frac{x+1}{5x+3} \leq 0 \text{ or } \frac{7x+3}{5x+3} \leq 0$$

$$x \in \left[-1, \frac{-3}{5}\right) \cup \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$[\alpha, \beta) \cup (\gamma - \delta]$$

$$\Rightarrow \alpha = -1,\ \beta = \gamma = \frac{-3}{5},\ \delta = \frac{-3}{7}$$

$$\therefore \left|3\alpha + 10\beta + 5\gamma + 21\delta\right| = \left|-3-9-9\right| = 21$$

Question: Image of (1, 2, 6) in plane containing (1, 4, 0), (1, 5, 1) and (0, 4, 1) is (α, β, γ). Find $\alpha^2 + \beta^2 + \gamma^2$.

Answer: 73.00

Solution:

Let $A(1,4,0),\ B(1,5,1), C(0,4,1)$

$$\overrightarrow{AB} = \hat{j} + \hat{k}$$

$$\overrightarrow{BC} - \hat{i} - \hat{j}$$

$$\Rightarrow \vec{n} = \overrightarrow{AB} \times \overrightarrow{BC}$$

$$= \left(\hat{j} + \hat{k}\right)\left(-\hat{i} - \hat{j}\right)$$

$$= \hat{k} - \hat{j} + \hat{i}$$

$$\Rightarrow\left[\vec{r}-\left(\hat{i}+4\hat{j}\right)\right]\cdot\vec{n}=0$$

$$\left[(x-1)\hat{i}+(y-4)\,\hat{j}+z\hat{k}\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=0$$

$$(x-1)-(y-4)+z=0$$

$$x-y+z+3=0$$

$$\Rightarrow\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-6}{1}=-2\left[\frac{1-2+6+3}{1^2+1^2+1^2}\right]$$

$$x-1=2-y=z-6=\frac{-2}{3}(8)$$

$$x=\frac{-16}{3}+1=\frac{-13}{3}=\alpha$$

$$y=2+\frac{16}{3}=\frac{22}{3}=\beta$$

$$z=\frac{-16}{3}+6=\frac{2}{3}=\gamma$$

$$\Rightarrow\alpha^2+\beta^2+\gamma^2=\frac{169+484+4}{9}$$

$$=\frac{657}{9}$$

$$=73$$

Question: If $\frac{2z-3i}{4z+2i}$ is real, then which is not possible?

Options:

(a) $x=0$

(b) $y+x^2+y^2\neq-\frac{1}{4}$

(c) $(x,y)=\left(0,-\frac{1}{2}\right)$

(d) $y\in\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$

Answer: (c)

Solution:

$$\frac{2z-3i}{4z+2i}=\frac{\overline{2z-3i}}{4z+2i}$$

$$\frac{2z-3i}{4z+2i}=\frac{2\bar{z}+3i}{4\bar{z}-2i}$$

$(2z-3i)(4\bar{z}-2i)=(2\bar{z}+3i)(4z+2i)$

$8z\bar{z}-4iz-12i\bar{z}+6i^2=8z\bar{z}+4i\bar{z}+12iz+6i^2$

$4i(z+\bar{z})+12i(z+\bar{z})=0$

$(z+\bar{z})(16i)=0$

$\Rightarrow z+\bar{z}=0$

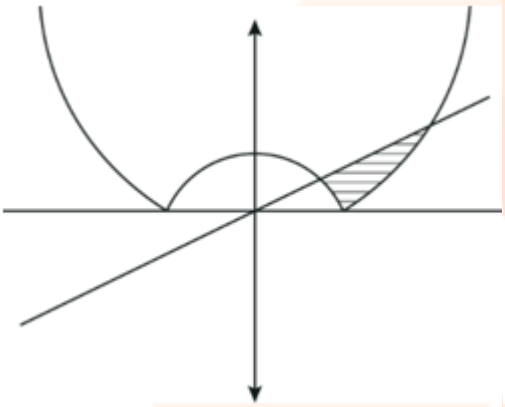
$2x=0$

$x=0$

Question: If the area under the curves $|x^2-2|\le y\le x$ is $6A+3\sqrt{2}$, then find A.

Answer: $\frac{9}{2}$

Solution:



$$A=\int_1^{\sqrt{2}} x-(2-x^2)\,dx+\int_{\sqrt{2}}^{2} x-(x^2-2)\,dx$$

$$A=\frac{x^2}{2}-2x+\frac{x^3}{3}\Big]_1^{\sqrt{2}}+\frac{x^2}{2}-\frac{x^3}{3}+2x\Big]_{\sqrt{2}}^{2}$$

$$=1-2\sqrt{2}+\frac{2\sqrt{2}}{3}-\frac{1}{2}+2-\frac{1}{3}+2-\frac{8}{3}+4-1+\frac{2\sqrt{2}}{3}-2\sqrt{2}$$

$$=\frac{4\sqrt{2}}{3}-4\sqrt{2}+\frac{9}{2}$$

$$=\frac{-8\sqrt{2}}{3}+\frac{9}{2}$$

Question: If $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4t^3}{3}$. Find $f\left(\frac{\pi^2}{4}\right)$.

Answer: $\pi - \frac{\pi^4}{16}$

Solution:

$$\int_0^{t^2} \left(f(x) + x^2 \right) dx = \frac{4t^3}{3}$$

Apply Leibnitz rule:

$$\left[f\left(t^2\right) + \left(t^2\right)^2 \right] \cdot 2t = 4t^2$$

$$f\left(t^2\right) + t^4 = 2t$$

$$f\left(t^2\right) = 2t - t^4$$

Put $t = \frac{\pi}{2}$

$$f\left(\frac{\pi^2}{4}\right) = 2\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{2}\right)^4$$

$$= \pi - \frac{\pi^4}{16}$$

Question: There are 3 cars, each having maximum capacity of 3 persons. In how many ways 8 persons can travel in these cars from place A to place B.

Answer: 1680.00

Solution:

8 persons can be divided into 3 groups

3, 3, 2

The number of ways $= \frac{8!}{3!3!2!} \times \frac{1}{2!} \times 3!$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 2 \times 2}$$

$$= 8 \times 7 \times 6 \times 5$$

$$= 56 \times 30$$

$$= 1680$$

Question: If $\int \left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x} \right) \ln x \, dx = \alpha \left(\frac{x}{e}\right)^{2x} + \beta \left(\frac{e}{x}\right)^{2x} + c$ where c is constant of integration, then

Options:

(a) $\alpha + \beta = 0$

(b) $\alpha+\beta=1$

(c) $\alpha\beta=\frac{1}{4}$

(d) $\alpha\beta=\frac{1}{2}$

Answer: (a)
Solution:

$$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx$$

Put $\left(\frac{x}{e}\right)^{2x}=t$

$$2x(\ln x-1)=\ln t$$

$$\left[2(\ln x-1)+2\right]dx=\frac{1}{t}\cdot dt$$

$$2\ln x\,dx=\frac{1}{t}\cdot dt$$

$$\Rightarrow\int\left(t+\frac{1}{t}\right)\frac{1}{2t}\cdot dt$$

$$=\frac{1}{2}\int dt+\frac{1}{2}\int\frac{1}{t^2}dt$$

$$=\frac{1}{2}t-\frac{1}{2t}+C$$

$$=\frac{1}{2}\left[\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{2x}\right]+C$$

Here $\alpha=\frac{1}{2},\beta=-\frac{1}{2}$

Hence $\alpha+\beta=\frac{1}{2}-\frac{1}{2}=0$

Question: Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum_{x\in S}\tan^2\left(\frac{x}{3}\right)$ then $\frac{1}{7}(\beta-14)^2$ is ____.
Answer: 32.00
Solution:

$$9^{1-\tan^2 x}+9^{\tan^2 x}=10$$

Put $9^{\tan^2 x} = t$

$$\frac{9}{t} + t = 10$$

$$9 + t^2 = 10t$$

$$t^2 - 10t + 9 = 0$$

$$t^2 - 9t - t + 9 = 0$$

$$t(t-9) - (t-9) = 0$$

$$(t-1)(t-9) = 0$$

$$t = 1 \text{ or } t = 9$$

$$9^{\tan^{-1} x} = 1 \quad \text{or} \quad 9^{\tan^2 x} = 9$$

$$\Rightarrow \tan^2 x = 0 \quad \text{or} \quad \tan^2 x = 1$$

$$\Rightarrow \tan x = 0 \quad \text{or} \quad \tan x = \pm 1$$

$$x = 0, \frac{\pi}{4}, -\frac{\pi}{4}$$

$$\Rightarrow \sum \tan^2\left(\frac{x}{3}\right) = \tan^2 0 + \tan^2\left(\frac{\pi}{12}\right) + \tan^2\left(\frac{-\pi}{12}\right)$$

$$= 0 + 2\tan^2\left(\frac{\pi}{12}\right)$$

$$= 2\tan^2 15°$$

$$= 2\left(2 - \sqrt{3}\right)^2$$

$$= 2\left(4 + 3 - 4\sqrt{3}\right)$$

$$= 2\left(7 - 4\sqrt{3}\right)$$

$$= 14 - 8\sqrt{3}$$

$$\Rightarrow \frac{1}{6}(\beta - 14)^2 = \frac{1}{6}\left(-8\sqrt{3}\right)^2$$

$$= \frac{8 \times 8 \times 3}{6}$$

$$= 32$$

Question: Let α be the remainder when $(22)^{2022} + (2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7, then $\alpha^2 + \beta^2$ is ____.

Answer: 5.00

Solution:

$(22)^{2022}+(2022)^{22}$

$(21+1)^{2022}+(2022)^{22}$

Here $(2022)^{22}$ is divisible by 3 as 2022 is divisible by 3

So, by expanding $(21+1)^{2022}$, we get

$(21+1)^{2022}={}^{2022}C_0(21)^{2022}+{}^{2022}C_1(21)^{2021}+\ldots{}^{2022}C_{2022}(1)^{2022}$

$=7\left((3)^{2022}(7)^{2021}+{}^{2022}C_1(3)^{2021}(7)^{2020}+\ldots\right)+1$

$=7k_1+1$

Remainder = 1

$(21+1)^{2022}+(2022)^{22}$

$(21+1)^{2022}+(2023-1)^{22}$

$7k_1+1+{}^{22}C_0(2023)^{22}-{}^{23}C_1(2023)^{21}+\ldots.-{}^{22}C_{21}(2023)(1)^{21}+{}^{22}C_{21}(1)^{22}$

$=7k_1+1+7\left({}^{22}C_0(7)^{21}(289)^{22}-{}^{22}C_1(7)^{20}(289)^{20}+\ldots.-{}^{22}C_4(289)\right)+1$

$=7k_1+1+7k_2+1$

$=7\mu+2$

Remainder = 2

$\alpha^2+\beta^2=1^2+2^2=5$

Question: A dice is rolled n times. If P(getting odd 7 times) is equal to P(getting even 9 times) and $P(\text{getting even 2 times})=\frac{k}{2^{15}}$. Find k.

Answer: 60.00
Solution:

P(getting 7 times odd) = P(getting 9 times even)

${}^nC_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7={}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

${}^nC_7={}^nC_9$

$n=7+9=16$

P(getting even 2 times) $={}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14}=60$

Question: $y=f(x)$ is a quadratic function passing through (-1, 0) and tangent to it at (1, 1) is $y=x$. Find x intercept by normal at point $(\alpha,\alpha+1),(\alpha>0)$.

Answer: 11.00

Solution:

Let $f(x)=ax^2+bx+(b-a)$

$f'(1)=2a+b=1$ and $f(1)=1=2b$

$a=\frac{1}{4},b=\frac{1}{2}$

$-\frac{dx}{dy}=\frac{-1}{2ax+b}=-\frac{2}{x+1}$

And $\alpha+1=\frac{1}{4}\alpha^2+\frac{1}{2}\alpha+\frac{1}{4}$

$4(\alpha+1)=(\alpha+1)^2$

$\Rightarrow \alpha+1=4$ or $\alpha=3$

Slope of normal $=-\frac{2}{4}=-\frac{1}{2}$

Equation of normal

$y-4=-\frac{1}{2}(x-3)$

For x intercept foot $y=0$

$-4=-\frac{1}{2}(x-3)$

$\Rightarrow x=11$

Question: $\sim(p\vee(\sim p\wedge q))$ is equivalent to

Answer:

Solution:

$\sim(p\vee(\sim p\wedge q))$

$=\sim(p\vee(p\vee\sim q))$

$=\sim p\wedge\sim(p\vee\sim q)$

$=\sim p\wedge(\sim p\wedge q)$

$=(\sim p\wedge\sim p)\wedge q$

$=\sim p\wedge q$

Question: Given sets $A=\{2,3,9\}$ and $B=\{4,8,12\}$. $(a_1,b_1)R(a_2,b_2)\Leftrightarrow \frac{a_1}{a_2}$ and $\frac{a_2}{b_1}$. Find number of elements in R.

Answer: 16.00

Solution:

When $a_1=a_2=2$

$a_1=2:\ b_2=4,8,12$
$a_2=2:\ b_1=4,8,12$ $\rightarrow$ 9 options

When $a_1=2, a_2=3$

$a_1=2: b_2=4,8,12$
$a_2=3:\ b_1=12$ $\rightarrow$ 3 options

When $a_1=3,\ a_2=1$, 3 options

When $a_1=3,\ a_2=3$

$b_1=12$
$b_2=12$ $\rightarrow$ 1 option

So total 16

Question: $A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$, $|adj\,(adj\ 2A)|=?$

Answer: 2^{16}

Solution:

$$A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$$

$|2A|^4=2^{12}\times|A|^4$

$$A=\begin{bmatrix}1 & 6 & 42\\ 1 & 7 & 56\\ 1 & 8 & 72\end{bmatrix}$$

$$=\begin{bmatrix}1 & 6 & 42\\ 0 & 1 & 14\\ 0 & 1 & 16\end{bmatrix}$$

$$|A|=2$$

$$|2A|^4=2^{12}\times|A|^4$$

$$=2^{12}\times 2^4$$

$$=2^{16}$$