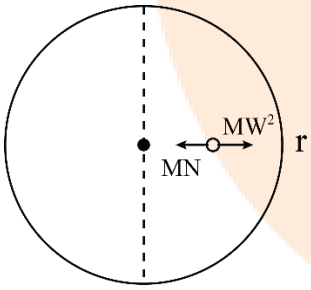
JEE-Mains-11-04-2023 [Memory Based] [Morning Shift]

Physics

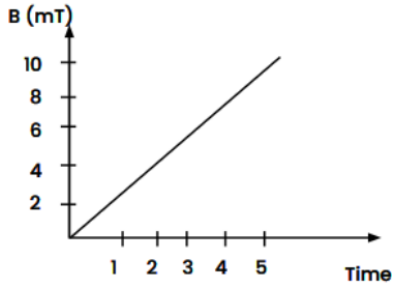
Question: Wave equation $x = 10^{-2} \sin(160t - 0.2x + \pi/4)$ find wave velocity
Options:
(a) 200 m/s
(b) 400 m/s
(c) 800 m/s
(d) 1600 m/s
Answer: (c)
Solution: $V = \frac{\omega}{K} = \frac{160}{0.2} = 800$

Question: Coin placed on a disc rotating with angular vel ω slips at a distance 1 cm from centre. At what distance it will slip when angular velocity is halved?
Options:
(a) 1 cm
(b) 2 cm
(c) 3 cm
(d) 4 cm
Answer: (d)
Solution:



$\mu N = m\omega^2 r$
So
$\frac{\mu N}{\mu N} = \frac{m\omega^2 (1)}{m\left(\frac{\omega}{2}\right)^2 .R}$
$x = 4$ cm

Question: If area of coil is 4 m^2 find EMF from 2 to 4 seconds



Options:

(a) 8×10^{-3} V

(b) 12×10^{-3} V

(c) 4×10^{-3} V

(d) 8×10^{-2} V

Answer: (a)

Solution:

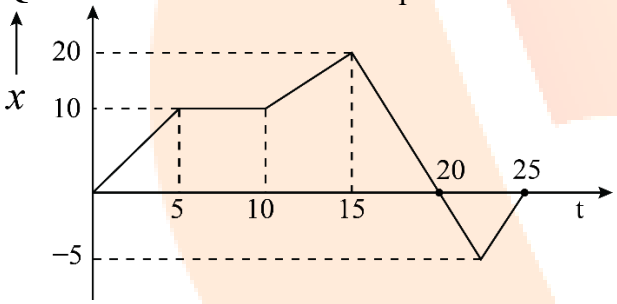
$\varepsilon = \left| A.\frac{dB}{dt} \right|$ and $\frac{dB}{dt} = 2\left(\frac{mT}{S}\right)$

So,

$\varepsilon = 4 \times 2 \times 10^{-3}$ T/s

$\varepsilon = 8 \times 10^{-3}$ V

Question: Find the ratio of displacement and distance.



Solution:

Positive area =

$$\left(\frac{1}{2} \times 5 \times 10\right) + (5 \times 10) + \frac{1}{2}(30)5 + \frac{1}{2} \times 5 \times 20$$

$$= 25 + 50 + 75 + 50 = 200$$

Negative area = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

Distance = positive area + negative area = $200 + \frac{25}{2} = \frac{425}{2}$

Displacement = positive area – negative area = $200 - \frac{25}{2} = \frac{375}{2}$

So ratio = $\frac{15}{17}$

Question: In photoelectric effect $V_{max,1} = V_0$ when wavelength is λ & $V_{max,2}$ is = $V_0/4$. When 2λ is used. Find work function.

Solution:

$$\frac{1}{2}\times\left(eV_0=\frac{hc}{\lambda}-\phi\right)$$

$$e\frac{V_0}{4}=\frac{hc}{2\lambda}-\phi$$

$(-)$

$$\frac{eV_0}{2}-\frac{eV_0}{4}=\frac{-\phi}{2}+\phi$$

$$\frac{eV_0}{4}=\frac{\phi}{2}\Rightarrow\phi=\frac{eV_0}{2}$$

Question: There are two identical resistances first they are joined in series then in parallel find ratio of heat produced in two cases assuming potential difference remains same

Options:

(a) 3 : 1

(b) 4 : 1

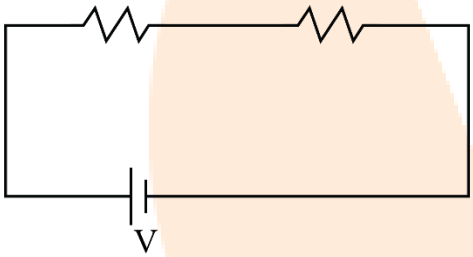
(c) 5 : 1

(d) 2 : 1

Answer: (b)

Solution:

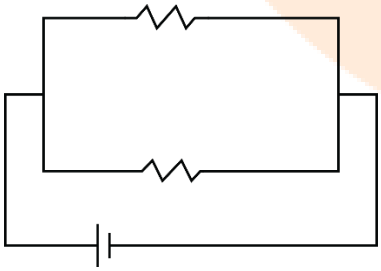
Case 1:



Heat = $2i^2Rt$

$$=2\left(\frac{V}{2R}\right)^2Rt=\left(\frac{V^2}{2R}.t\right)$$

Case 2:



$$(\text{Heat})_2=\frac{V^2}{(R/2)}.t$$

$$=\frac{2V^2}{R}.t$$

$$\text{So }\frac{(\text{Heat})_2}{(\text{Heat})_1}=\frac{2\frac{V^2}{R}.t}{\frac{V^2}{2R}.t}=\frac{4}{1}$$

Question: A planet has density 1/3 of density of earth & radius 4 time that of earth. find ratio of acceleration due to gravity on surface of planet & on Earth

Options:

(a) 4 : 3

(b) 2 : 3

(c) 5 : 7

(d) 1 : 1

Answer: (a)

Solution:

$$g_s = \frac{GM}{R^2} = \frac{G\rho\frac{4}{3}\pi R^3}{R^2}$$

$$g_s = \frac{4}{3}\pi G\rho R$$

$$\text{So } \frac{g_p}{g_E} = \frac{\frac{4}{3}\pi G\left(\frac{1}{3}\right)4}{\frac{4}{3}\pi G(1)(1)} = \frac{4}{3}$$

Question: A gun fires a bullet of mass 10 gm with vel 250 m/s as a result gun experiences a thrust force of 125 N find the number of bullets fired per second.

Answer: 50.00

Solution:

$$F = \left(\frac{NmU}{t}\right)$$

$$125 = \left(\frac{N}{t}\right).10\times10^{-3}\times250$$

$$\frac{N}{t} = \frac{1}{2}\times100 = 50$$

Question: A force $2 + 3x$ acts on particle at origin find work done from $x = 0$ to $x = 4$ (in J)

Answer: 32.00

Solution:

$$W = \int_{x_1}^{x_2} F\,dx = \int_0^4 (2+3x)\,dx = \left[2x + \frac{3}{2}x^2\right]_0^4$$

$$= (2\times4) + \left(\frac{3}{2}\times4^2\right) - 0 = 8 + 24 = 32$$

Question: Current sensitivity is increased by 25% by changing N and A and keeping R constant. Find the change in voltage sensitivity

Options:

(a) 25%

(b) 50%

(c) 12.5%

(d) 75%

Answer: (a)
Solution:

$$\left(\frac{\phi}{V}\right)=\frac{1}{R}\times\left(\frac{\phi}{i}\right)$$

So equal increase if R is constant
So 25%.

Question: A projectile is fired at an angle 30° with horizontal, it has same speed at t = 3 s and t = 5 s. find initial velocity
Options:
(a) 60 m/s
(b) 70 m/s
(c) 80 m/s
(d) 90 m/s
Answer: (c)
Solution:
Velocity in × doesn't change and velocity magnitude in Y is also given same θ = 30°

$$|V_y|=|V_y|$$

$$u\,\mathrm{Sin}\,\theta-g(3)=|u\,\mathrm{Sin}\,\theta-g(5)|$$

$$u\sin\theta-g3=g5-u\,\mathrm{Sin}\,\theta$$

$$2u\sin\theta=8g\Rightarrow u=80$$

Question: Statement 1: au, parsec, lightyear are units of length.
Statement 2: au < parsec < lightyear
Options:
(a) S1 - Correct, S2 - Correct
(b) S1 - Correct, S2 - False
(c) S1 - False, S2 - Correct
(d) S1 - False, S2 - False
Answer: (b)

Question: If height of transmitter antenna is 0 then find the receiver antenna in $x \times 10^{-2}$ such that Line of sight is 4 km
Answer: 125.00
Solution:

$$\text{Range}=0+\sqrt{2R_Eh_r}$$

$$4\times10^3=\sqrt{2\times64\times10^5\times x\times10^{-2}}$$

$$16\times10^6=64\times2\times10^3\times x$$

$$\frac{1000}{8}=x=125$$

Question: Capacitor charged to potential V has energy U_1 then its is connected to identical uncharged capacitor and final energy of system is U_2 find U_2/U_1
Options:
(a) 1 : 1
(b) 1 : 3

(c) 2 : 5
(d) 1 : 2
Answer: (d)
Solution:

$U_1 = \frac{1}{2}\frac{Q^2}{C}$ and finally change divides equally

$U_2 = 2 \times \left[\frac{1}{2}\frac{(Q/2)^2}{C}\right]$

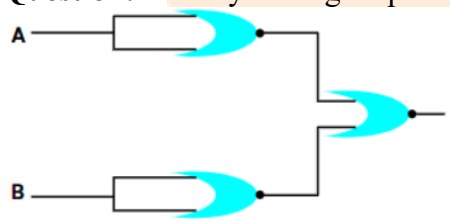
$U_2 = \frac{1}{2}\left[\frac{1}{2}\frac{QL}{C}\right]$

So $U_2 = \frac{1}{2}U_1 \Rightarrow \frac{U_2}{U_1} = 2$

Question: Half life of nucleus A is equal to mean life of nucleus B find the relation between λ_A & λ_B
Options:
(a) $\lambda_A = \ln 2 / \lambda_B$
(b) $\lambda_A = \ln 2\, \lambda_B$
(c) $\lambda_A = \lambda_B/2$
(d) $\lambda_A = 2\, \lambda_B$
Answer: (b)
Solution: $\frac{\ln(2)}{\lambda_A} = \frac{1}{\lambda_B}$

$\lambda_A = \ln 2 \lambda_B$

Question: Identify the logic operation of following circuit.



Options:
(a) AND
(b) OR
(c) NOR
(d) NAND
Answer: (a)

Question: If the magnetic moment of both coils A & B are equal then choose the correct relation if r_A = 10 cm, r_B = 20 cm, N_A = Number of turns of coil A, N_B = Number of turns of coil B, I_A = Current in coil A, I_B = Current in coil B
Options:
(a) $2N_AI_A = N_BI_B$
(b) $N_AI_A = N_BI_B$
(c) $N_AI_A = 4N_BI_B$
(d) $N_AI_A = 2N_BI_B$

Answer: (c)
Solution:

$M_1 = M_2$

$N_1 i_1 A_1 = N_2 i_2 A_2$

$N_1 i_1 \pi \left(10 \times 10^{-2}\right)^2 = N_2 i_2 \pi \left(20 \times 10^{-2}\right)^2$

$N_1 i_1 = 4 N_2 i_2$

Question: If light is passing through a medium of critical angle 45° then wave speed is
Options:

(a) $\frac{3}{\sqrt{2}} \times 10^8$

(b) $3\sqrt{2} \times 10^8$

(c) $\frac{3}{2} \times 10^8$

(d) 3×10^8

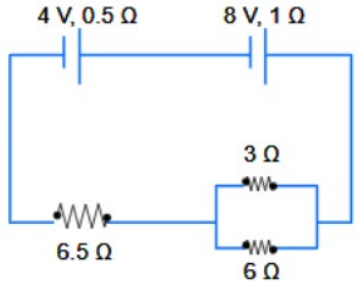
Answer: (a)
Solution:

$\sin^{-1}\left(\frac{1}{\mu}\right) = i_c = 45°$

$\frac{1}{\mu} = \frac{1}{\sqrt{2}} \Rightarrow \mu = \sqrt{2}$

So, $V = \frac{C}{\mu} = \frac{3 \times 10^8}{\sqrt{2}}$ m/s

Question: Find the current flowing in 3Ω resistor in the given circuit.



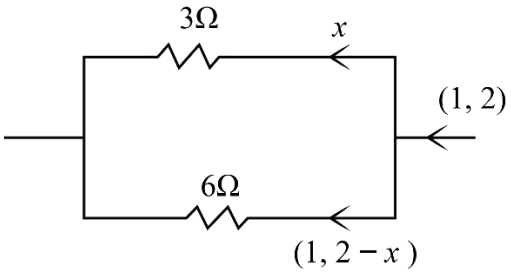
Options:

(a) 0.4 A
(b) 0.2 A
(c) 0.8 A
(d) 0.6 A

Answer: (c)
Solution:

$i = \frac{12}{0.5 + 1 + 6.5 + 2} = 1.2$

So,



$3x = (1.2 - x)6$

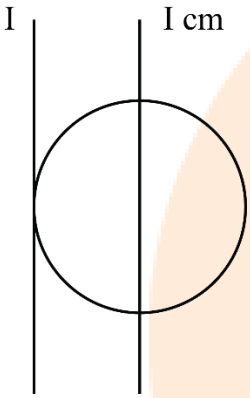
$x = 2.4 - 2x$

$x = 0.8$

Question: A solid sphere is rotating with ω = 10 rad s^{-1}. If I represents MOI about tangent to sphere & L represents angular moment about diameter & I = [$x \times 10^{-2}$] L. Find x ?

Answer: 35.00

Solution:



$I_{cm} = \frac{2}{5}mR^2$

$I = \frac{7}{5}mR^2$

So $\frac{I_{cm}}{I} = \frac{2}{7} \Rightarrow I_{cm} = \frac{2}{7}I$

$L = I\text{cm}\omega = \frac{2}{7}I.\omega$

$I = \frac{7}{2}\omega L = 35L$

Question: For a scale, melting point is –15°, boiling point is 65°, temp of –95° on this scale will represent what value on fahrenheit scale?

Options:

(a) 100°F

(b) –123°F

(c) 273°F

(d) –148°F

Answer: (d)

Solution:

$$\frac{x-(-15)}{65-(-15)}=\frac{F-32}{180}$$

but $x=-95$

$$\frac{-95+15}{80}=\frac{F-32}{180}$$

$$-180+32=F$$

$$\Rightarrow F=-148^\circ$$

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Chemistry

Question: Find the ratio of spin only magnetic moment of $Cr(CN)_6^{3-}$ and $Cr(H_2O)_6^{3+}$
Options:
(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) 2 : 3
Solution: $\mu = \sqrt{n(n+2)}$
Cr = $24(3d^5 4s^1)$, Cr^{+3} – $3d^3 4s^0$
Both CN^- and H_2O complex will have 3 unpaired electrons
∴ 1 : 1

Question: Ionisation Energy order of period second elements?
Li, Be, C, B, N, O, F
Options:
(a) Li > Be > B > C > N > F > O
(b) F > O > N > C > Be > B > Li
(c) F > O > N > C > B > Be > Li
(d) F > N > O > C > Be > B > Li
Answer: (d)
Solution: Half filled and fully filled are extra stable
F > N > O > C > Be > B > Li

Question: Statement-1: Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.
Statement-2: For Clean water the maximum prescribed concentration of zinc and Nitrate is 5 ppm each.
Options:
(a) Both statement 1 and statement 2 are correct
(b) both statement 1 and statement 2 are incorrect
(c) statement 1is incorrect and statement 2 is correct
(d) statement 1 is incorrect but statement 2 is correct
Answer: (a)
Solution: Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

Question: Match the species with correct shape

Column-I	**Column-II**
(A) NH_4^+	**(P)** Bent
(B) ClO_2^-	**(Q)** Linear

(C) SF_4	**(R)** Tetrahedral
(D) N_3^-	**(S)** See-saw

Options:
(a) A – R; B – P; C – S; D – Q
(b) A – Q; B – P; C – R; D – S
(c) A – S; B – P; C – Q; D – R
(d) A – Q; B – P; C – S; D – R
Answer: (a)
Solution: Check hybridization for shapes

Question: Which of the following is correct set of non-ambidentate ligand?
Options:
(a) NO_2, EDTA
(b) SCN-, NO_2
(c) C_2O_4, H_2O
(d) SCN-, CN-
Answer: (c)
Solution: Fact based.

Question: 25% of 250g sugar solution & 40% of 500g sugar solution are mixed then find out the mass percentage in the solution
Options:
(a) 35

(b) 45

(c) 40

(d) 38

Answer: (a)
Solution: $25 \times 250 + 40 \times 500 = x \times 750$

$x = 35$

Question: 0.004M solution of K_2SO_4 is isotonic with 0.01 M of glucose. Find degree of dissociation of K_2SO_4,
Options:
(a) 75%
(b) 25%
(c) 50%
(d) 85%
Answer: (a)
Solution: $\pi_1 = \pi_2$
For isotonic
$\therefore 0.004 \times i \times RT = 0.01\ RT$

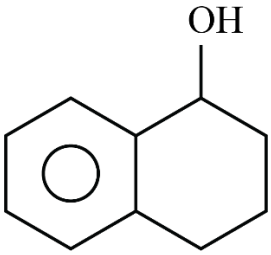
$$i = \frac{0.01}{0.004} = 2.5$$

$$\alpha = \frac{i-1}{n-1}$$

$\alpha = 0.75$

Question: Find correct order of electrophilic aromatic substitution reaction

a)



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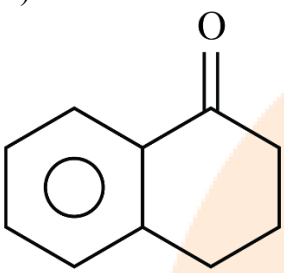

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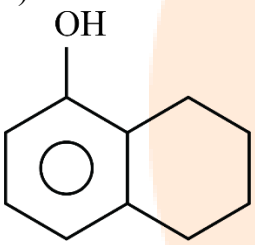
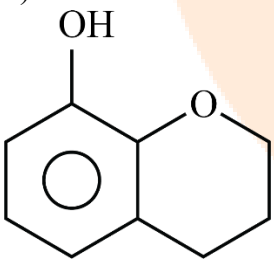


d)



Options:

(a) d > c > a > b

(b) c > d > b > a

(c) d > c > b > a

(d) a > b > c > d

Answer: (a)

Solution: EWG increases EAS reaction.

Question: Statement-1: CH_4 and H_2O in presence of Ni Catalyst produces H_2 gas.

Statement-2: Sodium Nitrite reacts with NH_4Cl gives H_2, N_2 and H_2O

Options:
(a) Both statement 1 and statement 2 are correct
(b) Both statements are incorrect
(c) Statement 1 is correct and statement 2 is incorrect
(d) Statement 1 is incorrect and statement 2 is correct
Answer: (a)
Solution: In the laboratory, dinitrogen is prepared by treating an aqueous solution of ammonium chloride with sodium nitrate.

$NH_4Cl(aq) + NaNO_2(aq) \rightarrow N_2(g) + 2H_2O(l) + NaCl\ (aq)$

e.g.,

$$CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270K} CO(g) + 3H_2(g)$$

Question: If electrode potentials are

$Pb^{2+}/Pb = m$
$Pb^{4+}/Pb = n$
Then Find the value of x in?
$Pb^{2+}/Pb^{4+} = m\text{-}xn$
Options:
(a) 2
(b) 3
(c) 4
(d) 1
Answer: (a)
Solution: $\Delta G_1^0 = -2Fm$, $\Delta G_2^0 = -4Fn$

$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$ Thus (m–2n)

Question: To 25 ml of 1M silver nitrate 1.05 M potassium iodide is added dropwise. In

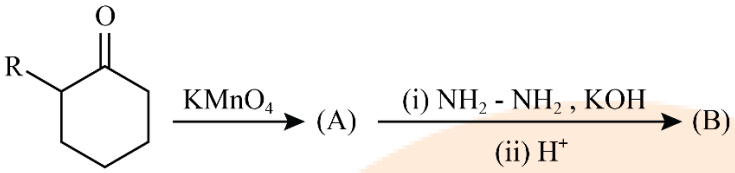
colloidal Sol formed fixed and diffused layer consist of respectively:

Options:
(a) NO_3^-, Ag^+
(b)Ag+, K^+
(c) Ag+, I^-
(d) Ag+, NO_3^-
Answer: (d)
Solution: Fixed layer of Ag^+ on AgI and mobile layer of NO_3^-

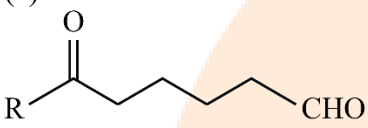
Question: A Solutions contain 2 salts anions,
Statement-1: Solution on treatment with freshly prepared $FeSO_4$ gave a brown ring.
Statement-2: On reaction with $FeCl_3$ and boiling gave a reddish-brown precipitate?
The two anions are :

Options:
(a) nitrate and acetate
(b) nitrite and sulphate
(c) nitrite and oxalate
(d) nitrate and phosphate
Answer: (a)
Solution: Fact based

Question:



B is
Options:
(a)



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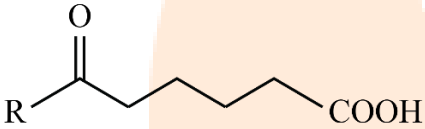

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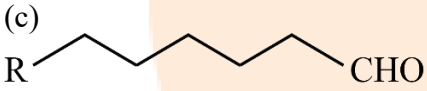
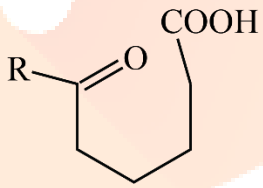


(d)

Answer: (d)

Solution: Popoff rule is followed to form A



and Wolf Kishner's reduction of carbonyl is done

Question: Which type of copper is formed by the following reactions?
$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$
$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$
Options:
(a) Blister copper
(b) Copper crisp
(c) Reduced copper
(d) Copper slag
Answer: (a)
Solution: Fact based

Question: Identify the correct statement about the compound $GaAlCl_4$
Options:
(a) Chlorine atom is bonded to both Ga and Al
(b) Ga is cationic part and less electronegative than Al
(c) Chlorine atom forms co – ordinate bond with Ga
(d) Chlorine atom is bonded to Al
Answer: (d)
Solution: Fact based.

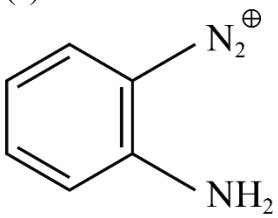
Question: In a container at constant temperature correct RMS velocity of the following.
Options:
(a) $Ne > Cl_2 > UF_6$
(b) $Cl_2 > Ne > UF_6$
(c) $UF_6 > Ne > Cl_2$
(d) $UF_6 > Cl_2 > Ne$
Answer: (a)
Solution:

$$u_{rms} = \sqrt{\frac{3RT}{M}}$$

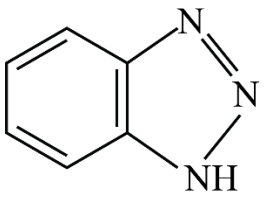
∴ RMS is inversely proportional to molecular mass ($UF_6 > Cl_2 > Ne$)

Question: Which of the following can be represented as a meridional isomer?
Options:
(a) $[Pt(NH_3)_3Cl_3]^+$
(b) $[Pt(en)_3]^{4+}$
(c) $[Pt(en)_2Cl_2]^{2+}$
(d) $[Pt(en)_2(NH_3)_2]^{4+}$
Answer: (a)
Solution: Fact based.

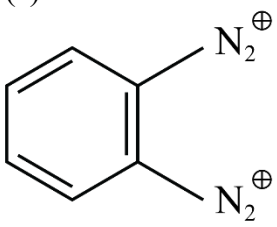
Question: o-phenylenediamine $\xrightarrow{HNO_2}$
Options:
(a)



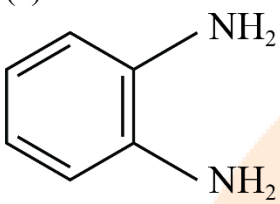
(b)



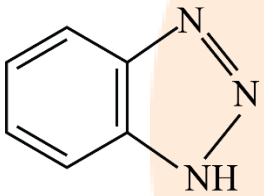
(c)



(d)



Answer: (b)
Solution:



JEE-Mains-11-04-2023 [Memory Based] [Morning Shift]

Mathematics

Question: $z_1 = 5+2i$ is rotated by $90°$ anticlockwise to get ω_1, and $z_2 = 3+i$ is rotated $90°$ clockwise to get ω_2. $\arg(\omega_1 - \omega_2) = ?$

Answer: $\pi - \tan^{-1}\left(\frac{8}{3}\right)$

Solution:

$\omega_1 = (5+2i)i = 5i-2$

$\omega_2 = (3+i)(-i) = -3i+1$

$\omega_1 - \omega_2 = 8i-3 = -3+8i$

$\arg(\omega_1 - \omega_2) = \arg(-3+8i)$

$= \pi - \tan^{-1}\left(\frac{8}{3}\right)$

Question: Number of integral terms in $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is

Answer: 171.00
Solution:

$T_{k+1} = {}^{680}C_k (3)^{\frac{680-k}{2}} (5)^{\frac{k}{4}}$

$k = 0, 4, 8, ..., 680$

Number of integral terms = $\frac{680}{4} + 1 = 171$

So 171 terms

Question: In the expansion of $(2+x)^9$, find the mean of the coefficients of $x, x^2, x^3, ...x^7$.

Answer: 2736.00
Solution:

$${}^9C_1 \times 2^8 + {}^9C_2 \times 2^7 + ... + {}^9C_7 2^2$$

$$\frac{3^9 - 2^9 - 9 \times 2 - 1}{7} = \frac{3^9 - 2^9 - 19}{7}$$

Question: $M = (m_{ij}); m_{ij} \in \{0,1,2\}, i \geq 1, j \leq 2$. $A = \{m : M \text{ is invertible}\}$. Number of $n(A)$.

Answer: 52.00

Solution:

Order: 1×1, $M = (1)$ or $(2) \to 2$ matrices

Order: 2×2, $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

$ad = bc = 0$, No. of matrices $= 5 \times 5 = 25$

$ad = bc = 1$, No. of matrices $= 1 \times 1 = 1$

$ad = bc = 2$, No. of matrices $= 2 \times 2 = 4$

$ad = bc = 4$, No. of matrices $= 1 \times 1 = 1$

No. of invertible matrix of order 2×2

$= 3^4 - 25 - 1 - 4 - 1 = 50$

Total $= 50 + 2 = 52$

Question: Five students given roll number and to be seated according to it. Find ways such that no students is on correct seat.

Answer: 44.00

Solution:

Number of seating arrangements

$$5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$$

$$= 5!\left(\frac{120 - 120 + 60 - 20 + 5 - 1}{120}\right)$$

$$= 44$$

Question: a and b are the roots of $x^2 - 7x - 1 = 0$, then $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}} = ?$

Answer: 51.00

Solution:

Given $x^2-7x-1=0$

$a^2-1=7a$

$a^4+1=49a^2+2a^2$

$a^4+1=51a^2$

$$\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}=\frac{a^{17}\left(51a^2\right)+b^{17}\left(51b^2\right)}{a^{19}+b^{19}}=51$$

Question: Number of ordered truth values of (p,q,r) such that $(p\vee q)\wedge(p\vee r)\rightarrow(q\vee r)$ is true.

Answer: 7.00

Solution:

False only if

$(p\vee q)\wedge(p\vee r)$ is T & $q\vee r$ is F

$$q=r=F$$

So p has to be true

In remaining 7 cases it is True

Question: $x+y+z=15$. Find number of ordered solutions (x,y,z) such that $x,y,z\geq 0$

Answer: 136.00

Solution:

Total ordered solutions

$\Rightarrow {}^{15+3-1}C_{3-1}={}^{17}C_2$

$\Rightarrow {}^{15+2}C_2=136$

Question: Area of region $(x,y):x^2+(y-2)^2\leq 4;\ x^2\geq 2y$ is

Options:

(a) $\pi-\frac{8}{3}$

(b) $\pi+\frac{8}{3}$

(c) $2\pi-\frac{16}{3}$

(d) $2\pi+\frac{16}{3}$

Answer: (a)

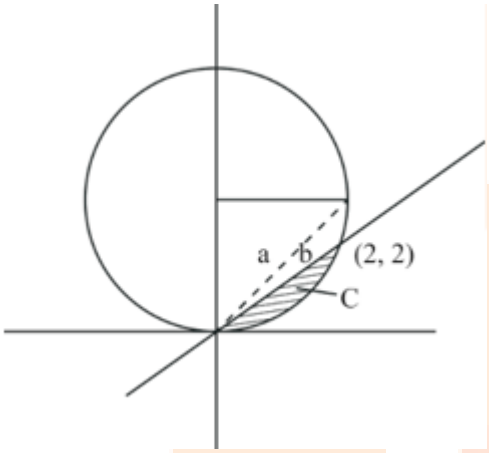
Solution:

$x^2 = 2y$

$2y + (y-2)^2 = 4$

$y^2 - 2y = 0$

$y = 0, 2$



$a + b + c = \frac{\pi}{4} \cdot 4 = \pi$

$a = \frac{1}{2} \cdot 2 \cdot 2 = 2$

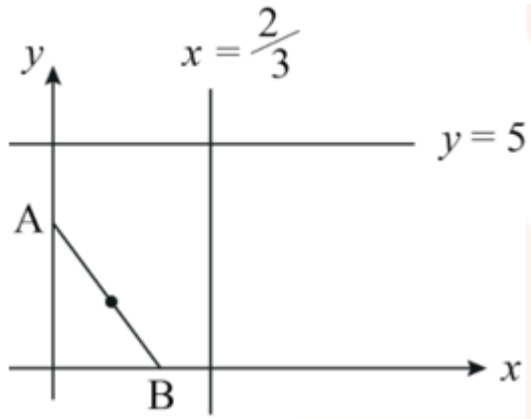
$b = \frac{8}{6 \cdot 2} = \frac{2}{3}$

$c = \pi - 2 - \frac{2}{3} = \pi - \frac{8}{3}$

Question: The lines $y = 0,\ x = 0, y = 5, x = \frac{2}{3}$ forms a rectangle. $A(0, a)$ and $B(b, 0)$ are points such that AB divides area of rectangle in $1:4$. Midpoint of AB lies on

Answer: $xy = \frac{1}{3}$

Solution:



$A_R = \frac{10}{3}$

$A_\Delta = \frac{ab}{2} = \frac{2}{3}$

$ab = \frac{4}{3}$

$\left(\frac{b}{2}, \frac{a}{2}\right)$ lies on $xy = \frac{1}{3}$

Question: Number of elements in set $S = \left[\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\right]$ is

Answer: 9.00

Solution:

$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

Writing in x terms

$\left(3\cos^4 x - 3\cos^2 x\right) - 2\sin^6 x + 2\sin^2 x = 0$

$3\cos^2 x\left(\cos^2 x - 1\right) - 2\sin^2 x\left(-\sin^4 x + 1\right) = 0$

$3\cos^2 x \times \sin^2 x = 2\sin^2 x \times \cos^2 x\left(1 + \sin^2 x\right)$

$\sin^2 x = 0 \rightarrow 3$ solutions

$\cos^2 x = 0 \rightarrow 2$ solutions

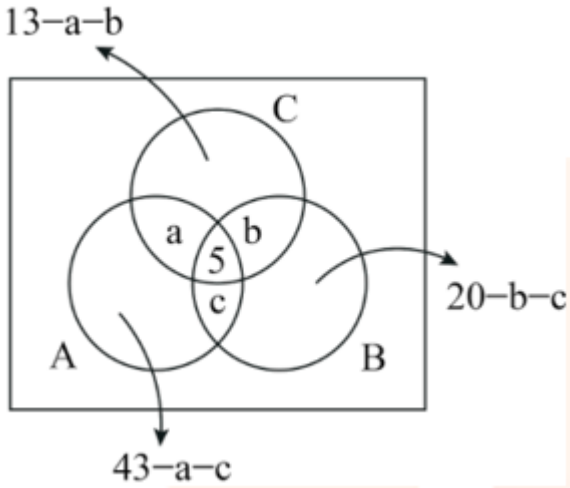
$\sin^2 x = \frac{1}{2} \rightarrow 4$ solutions

So total 9 solutions.

Question: 48 medals were given in event A. 25 medals were given in event B and 18 medals were given in event C. Total men = 60. 5 men got medals in all 3 events. How many got medals in exactly 2 events.

Answer: 21.00

Solution:



$76-2(a+b+c)+(a+b+c)+5=60$

$\Rightarrow a+b+c=21$

Question: $A^T=\alpha A-I$, $\left|A^2-A\right|=4$. Find sum of all possible values of α. Order of A is 2.

Answer: 4.00

Solution:

For $n=\alpha$,

$A=\alpha A^T-I$

$A^T=\alpha A-I$

$A=\alpha(\alpha A-I)-I$

$A=\alpha^2 A-(\alpha+1)I$

$A=\dfrac{-(\alpha+1)}{1-\alpha^2}I=\dfrac{I}{\alpha-1}$

$\left|A^2-A\right|=|A||A-I|$

$=\left(\dfrac{1}{\alpha-1}\right)^n\left(\dfrac{2-\alpha}{\alpha-1}\right)^n=4$

$(2-\alpha)^2=4(\alpha-1)^4$

$=4\left[\alpha^4-4\alpha^3+1\right]$

Sum of α = 4

Question: $\left(1-x^2y^2\right)dx = x\,dy + y\,dx$. $y(1)=2$, $y(2)=\alpha$. Find α.

Answer:

Solution:

$$\left(1-x^2y^2\right)dx = d\left(xy\right)$$

$$\int dx = \int \frac{dxy}{1-x^2y^2}$$

$$x = \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| + C$$

$$(1,2)$$

$$1 = \frac{1}{2}\ln|-3| + C$$

$$C = \frac{2}{\ln 3}$$

Question: $f(x) = \left[x^2 - x\right] + \left[\left|[x]-x\right|\right]$ check continuity at $x = 0,1$

Answer: Discontinuous at $x = 1$

Solution:

$$f(x) = \left[x^2 - x\right] + \{x\}$$

$$f(x) = \left[x^2 - x\right]$$

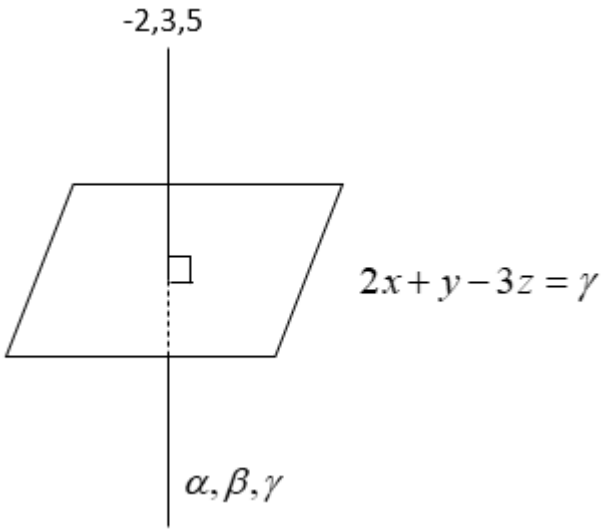
$$\lim_{x\to 0^+} f(x) = -1,\ \lim_{x\to 0^-} f(x) = 0$$

$$\lim_{x\to 1^+} f(x) = 0,\ \lim_{x\to 1^-} f(x) = -1$$

Question: Consider the plane $2x + y - 3z = 6$. If (α,β,γ) is the image of point $(-2,3,5)$ in the given plane, then $\alpha+\beta+\gamma =$ ___

Answer: 0.00

Solution:



$$\frac{\alpha+2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=\frac{-2(-22)}{14}$$

By solving

$$\alpha+\beta+\gamma=0$$

Question: $x_1, x_2, ...x_{100}$ are in A.P. If $x_1=2,\ \bar{x}=200,\ y_i=i(x_i-1), i\in\{1,2,...100\}$, then $\bar{y}=?$

Answer: $\frac{26765}{2}$

Solution:

$2, x_2, x_3, ...x_{100}$ are in AP

$$\frac{\frac{100}{2}(4+99\times d)}{100}=200$$

$$d=4$$

$$y_i=i(2+(i-1)d)-1$$

$$y_i=i(2+(i-1)4)-1$$

$$y_i=i(4i-3)=4i^2-3i$$

$$\bar{y}=\frac{4\sum i^2-3\sum i}{100}$$

$$=\frac{4\times\frac{100\times101\times201}{6}-3\times\frac{100\times101}{2}}{100}$$

$$=\frac{26765}{2}$$

Question: If $\log_{x+\frac{7}{2}}\left(\frac{x+7}{2x+3}\right)^2 \geq 0$, then total number of integral solutions is/are ___

Answer: 7.00

Solution:

Case 1: when base is $0<\left(x+\frac{7}{2}\right)<1$ and $\left|\frac{x+7}{2x+3}\right| \leq 1 \Rightarrow -1 \leq \frac{x+7}{2x+3} \leq 1$

(i) $\frac{x+7}{2x+3} \leq 1 \Rightarrow \frac{x+7}{2x+3} \leq 1$

$x \in \left(-\infty, -\frac{3}{2}\right) \cup [4, \infty]$

Or

(ii) $-1 \leq \frac{x+7}{2x+3} \Rightarrow \frac{3x+10}{2x+3} \geq 0$

$x \in \left(-\infty, -\frac{10}{3}\right] \cup \left[-\frac{3}{2}, \infty\right)$

Required solutions for case 1 $x \in \left(-\frac{7}{2}, -\frac{5}{3}\right)$

Case 2: when base is $\left(x+\frac{7}{2}\right)>1$ and

$\left|\frac{x+7}{2x+3}\right| \geq 1 \Rightarrow \frac{x+7}{2x+3} \geq 1$ or $-1 \geq \frac{x+7}{2x+3}$

(i) $\frac{x+7}{2x+3} \geq 1 \Rightarrow \frac{x-4}{2x+3} \leq 0$

$x \in \left(-\frac{3}{2}, 4\right)$

Or

(ii) $-1 \geq \frac{x+7}{2x+3} \Rightarrow \frac{3x+10}{2x+3} \leq 0$

$x \in \left(-\frac{10}{3}, -\frac{3}{2}\right) \cup \left(-\frac{5}{2}, -\frac{3}{2}\right)$

Required solutions for case 2

$$x \in \left(-\frac{5}{2}, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, 4\right]$$

Total number of integral solutions = $\pm 2, \pm 1, 0, 3, 4$

Question: Find the value of integral $\int_{-\log e^2}^{\log e^2} e^x \left(\log_e \left(e^x + \sqrt{1+e^{2x}}\right) dx\right)$ is

Answer:
Solution:

$$\int_{-\log e^2}^{\log e^2} e^x \left(\log_e \left(e^x + \sqrt{1+e^{2x}}\right) dx\right)$$

Put $e^x = t$

$$\int_{\frac{1}{2}}^{2} \ln\left(t + \sqrt{1+t^2}\right) dt$$

$$t \ln\left(t + \sqrt{1+t^2}\right)\Big|_{\frac{1}{2}}^{2} - \int_{\frac{1}{2}}^{2} \frac{t}{\sqrt{1+t^2}}$$

$$2\ln\left(2+\sqrt{5}\right) - \frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right) - \frac{1}{2} 2 \times \sqrt{1+t^2}\Big|_{\frac{1}{2}}^{2}$$

$$2\ln\left(2+\sqrt{5}\right) - \frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right) - \left(\sqrt{5} - \frac{\sqrt{5}}{2}\right)$$