PART : MATHEMATICS

1. If $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$ then the value of |adj (adj(2A))| is equal to

(1) 2^{16} (2) 2^{14} (3) 2^{20} (4) 2^{8}

Ans. **(1)**

Sol. $|A| = \frac{1}{5!6!7!} \times 5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$

$|A| = \begin{vmatrix} 1 & 6 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} \Rightarrow |A| = 2$

$|\text{Adj (adj (2A))}| = |2A|^{(n-1)^2} = |2A|^4 = 2^{12}|A|^4 = 2^{16}$

2. The sum of all four digit numbers formed by 1, 2, 2, 3 is

Ans. **(26664)**

Sol. Sum of all 4 digit numbers

= sum of digit at unit place × 10^0

+ sum of digit at term place × 10^1

+ sum of digit at hundred place × 10^2

+ sum of digit at thousand place × 10^3

$= \left(1 \times \frac{3!}{2!} + 2 \times 3! + 3 \times \frac{3!}{2!}\right)(10^0 + 10^1 + 10^2 + 10^3)$

= 24 × (1111)

= 26664

3. If coefficient of x and x^2 in the expansion of $(1 + x)^p (1 - x)^q$ are 4 and – 5 respectively then the value of 2p + 3q is

(1) 61 (2) 62 (3) 63 (4) 64

Ans. **(3)**

Sol. $(1 + x)^p (1 - x)^q = (1 + {}^pC_1x + ...)(1 - {}^qC_1x ....)$

coefficient of $x^1 = {}^pC_1 - {}^qC_1 = p - q = 4$ ......(1)

coefficient of $x^2 = -{}^pC_1\,{}^qC_1 + {}^pC_2 + {}^qC_2$

$= -p.q + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5 \Rightarrow (p-q)^2 - p - q = -10 \Rightarrow p + q = 26$ ......(2)

by (1) and (2) $\Rightarrow$ p = 15 and q = 11 $\Rightarrow$ 2p + 3q = 63

4. If domain of $\sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (r\ \delta]$ then the value of $|3\alpha + 10(\beta + r) + 21\delta|$ is

(1) 21 (2) 22 (3) 33 (4) 24

Ans. (4)

Sol. $\frac{2x}{5x+3} \le -1$ or $\frac{2x}{5x+3} \ge 1$

$\frac{2x}{5x+3} + 1 \le 0$ or $\frac{2x}{5x+3} - 1 \ge 0$

$\frac{7x+3}{5x+3} \le 0$ or $\frac{3(x+1)}{5x+3} \le 0 \Rightarrow x \in [-1, -3/5) \cup (-3/5, -3/7]$

$\alpha = -1$, $\beta = -3/5$, $\gamma = -3/5$, $\delta = -3/7$

$\left|3(-1) + 10\left(-\frac{6}{5}\right) + 21\left(-\frac{3}{7}\right)\right| = |-3 - 12 - 9| = 24$

5. If $9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then the value of $\sum \tan^2\left(\frac{x}{3}\right)$

(1) $8\sqrt{3}$ (2) 14 (3) $14 - 8\sqrt{3}$ (4) $14 + 8\sqrt{3}$

Ans. (3)

Sol. $9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$

$\Rightarrow \frac{9}{y} + y = 10$ (where $9^{\tan^2 x} = y$)

So, $9 + y^2 = 10y$

$y^2 - 10y + 9 = 0$

$(y-1)(y-9) = 0$

So, $9^{\tan^2 x} = 1$ & $9^{\tan^2 x} = 9$

$\tan^2 x = 0$ $\qquad$ $\tan^2 x = 1$

$\downarrow$ $\qquad$ $\swarrow$ $\searrow$

$x = 0$ $\qquad$ $\tan x = 1$ $\qquad$ $\tan x = -1$

$\downarrow$ $\qquad$ $\downarrow$

$x = \frac{\pi}{4}$ $\qquad$ $x = -\frac{\pi}{4}$

Now $\sum \tan^2\frac{x}{3} = \tan^2\frac{0}{3} + \tan^2\frac{\pi}{12} + \tan^2\frac{\pi}{12}$

$= 2\tan^2\frac{\pi}{12}$

$= 2(2-\sqrt{3})^2$

$= 2(7-4\sqrt{3})$

$= 14 - 8\sqrt{3}$

6. If $(22)^{2022} + (2022)^{22}$ is divided by 3 gives remainder α and when divided by 7 gives remainder β then the value of $\alpha^2 + \beta^2$ is

(1) 7 (2) 5 (3) 8 (4) 9

Ans. (2)

Sol. $(22)^{2022} + (2022)^{22}$

$(21 + 1)^{2022} + (2022)^{22}$

$= 21\lambda + 1 + 3\mu \Rightarrow \alpha = 1$

Now $(21 + 1)^{2022} + (2023 - 1)^{22}$

$= 21\lambda + 2023\mu + 2 \Rightarrow \beta = 2$

Now, $\alpha^2 + \beta^2 = 1 + 4 = 5$

7. If p, q, r are any three statements then $\sim(p \vee (\sim p \wedge q))$ is equivalent to

(1) $\sim(p \vee q)$ (2) $p \vee q$ (3) $\sim p \vee \sim q$ (4) $p \wedge \sim q$

Ans. (1)

Sol. $\sim(p \vee (\sim p \wedge q)) = \sim((p \vee \sim p) \wedge (p \vee q))$

$\sim[t \wedge (p \vee q)]$

$= \sim(p \vee q)$

8. $S_n = 4 + 11 + 21 + 34 + 50$ ....

Find $\frac{1}{60}(S_{29} - S_9)$

Ans. (223)

Sol. $S_n = 4 + 11 + 21 + 34 + 50$ ....

Difference $\Rightarrow$ 7 10 13 16

So, $T_n = 4 + \frac{n-1}{2}(2 \times 7 + (n-1-1)3) = \frac{3n^2 + 5n}{2}$

So, $S_n = \sum T_n = \frac{1}{2}(3\Sigma n^2 + 5\Sigma n)$

$= \frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} + 5\frac{n(n+1)}{2}\right) = \frac{n(n+1)(n+3)}{2}$

So, $\frac{1}{60}(S_{29} - S_9)$

$= \frac{1}{60}\left(\frac{29.30.32}{2} - \frac{9.10.12}{2}\right) = 223$

9. If circumcentre and orthocentre of Δ ABC are P and Q respectively then $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is equal to

(1) $2\overrightarrow{PQ}$ (2) $2\overrightarrow{QP}$ (3) $\overrightarrow{PQ}$ (4) $\overrightarrow{QP}$

Ans. (3)

Sol. Let position vectors of vertices of Δ ABC be A $(\vec{a})$, B $(\vec{b})$ and C $(\vec{c})$

Now

centroid G $\left(\frac{\vec{a} + \vec{b} + \vec{c}}{3}\right)$

Now if circumcentre of & orthocentre of Δ ABC are $(\vec{p})$ and $Q(\vec{q})$ then

$G\left(\frac{2\vec{p}+\vec{q}}{3}\right)$

Now $\frac{2\vec{p}+\vec{q}}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$2\vec{p}+\vec{q} = \vec{a}+\vec{b}+\vec{c}$ _______(1)

Now $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC} = \vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}$

$= \left(2\vec{p}+\vec{q}\right) - 3\vec{p} = \vec{q}-\vec{p} = \overrightarrow{PQ}$

10. In an AGP $a_1, a_2, 2, a_3, a_4$; If the common ratio is 2 & sum is $\frac{49}{2}$ then the value of a_4 is

(1) 15 (2) 16 (3) 17 (4) 18

Ans. **(2)**

Sol.

a_1	a_2	2	a_3	a_4
↓	↓	↓	↓	↓
a	$2(a+d)$	$4(a+2d)$	$8(a+3d)$	$16(a+4d)$

So $4(a+2d) = 2$

$2a+4d = 1$ ......(1)

Sum $= a+2a+2d+4a+8d+8a+24d+16a+64d$

$= 31a+98d = \frac{49}{2}$ ....(2)

by equation (1) and (2) $\Rightarrow a = 0$ and $d = \frac{1}{4}$

So, $a_4 = 16(a+4d) = 16$

11. If probability of getting 7 odd numbers is equal to probability of getting 9 odd number in throwing of die. Find k if probability of getting even numbers is $\frac{K}{2^{17}}$.

(1) 1 (2) 2 (3) 4 (4) 8

Ans. **(2)**

Sol. ${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

So ${}^nC_7 = {}^nC_9$

So $n = 16$

So, Probability of getting even numbers

is $\left(\frac{1}{2}\right)^{16} = \frac{K}{2^{17}}$

So $K = 2$.

12. If line passing through (1,2) cuts the circle $x^2 + y^2 = 16$ at A and B then locus of the mid points of AB is.

(1) $x^2 + y^2 + x + y = 0$ (2) $x^2 + y^2 - x - y = 0$

(3) $x^2 + y^2 + 2x - y = 0$ (4) $x^2 + y^2 - x - 2y = 0$

Ans. (4)

Sol. Equation of chord whose mid point is P (h,k) is $T = S_1$

$\Rightarrow xh + yk = h^2 + k^2$

it passes through (1,2)

$h + 2k = h^2 + k^2$

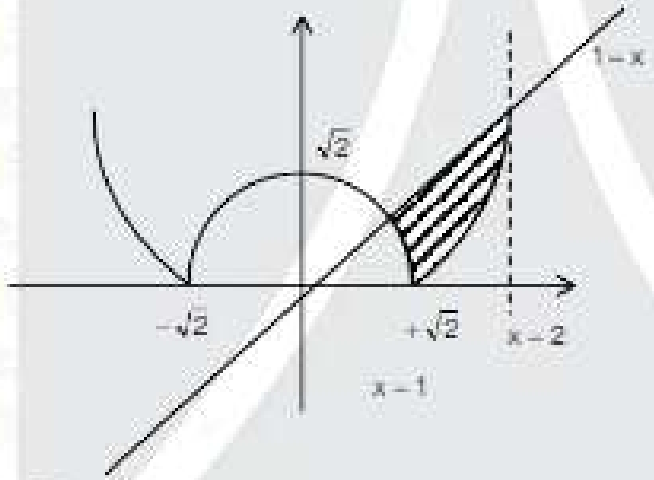
locus is : $x^2 + y^2 - x - 2y = 0$

13. Let A be the area $|x^2 - 2| \le y \le x$ Then $6A + 16\sqrt{2}$ is

(1) 26 (2) 27 (3) 28 (4) 29

Ans. (2)

Sol. $= |x^2 - 2| \le y \le x$



So Area $\int_1^{\sqrt{2}} \left(x - (2 - x^2)\right)dx + \int_{\sqrt{2}}^{2} \left(x - (x^2 - 2)\right)dx$

$= \left[\frac{x^2}{2} - 2x + \frac{x^3}{3}\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^{2}$

$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$

$= 1 - \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} - 1 - \frac{4\sqrt{2}}{3}$

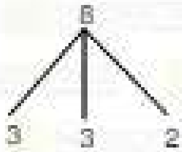
$A = \frac{9}{2} - \frac{8\sqrt{2}}{3}$

So $6A + 16\sqrt{2} = 27$

14. The number of ways in which 8 persons can go from A to B in 3 cars such that 1 car can accommodate at most 3 persons is

Ans. **(1680)**

Sol.



$\frac{8!}{3!\,3!\,2!\,2!}\,3!$

$= 1680$

15. The value of $\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx$ is

(1) $\frac{1}{2}\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)+C$

(2) $\frac{1}{2}\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{2x}\right)+C$

(3) $\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right)+C$

(4) $\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{-2x}\right)+C$

Ans. **(2)**

Sol. Put $\left(\frac{x}{e}\right)^{x}=t$

So, $\ell nt = x\cdot\ell n\left(\frac{x}{e}\right)$

$= x(\ell nx - 1)$

$\Rightarrow \frac{dt}{t} = \ell nx\,dx$

So, $\int\left(t^2+\frac{1}{t^2}\right)\frac{dt}{t}$

$= \int\left(t+\frac{1}{t^3}\right)dt$

$\frac{t^2}{2}+\frac{t^{-2}}{-2}+C$

$= \frac{1}{2}\left(t^2-t^{-2}\right)+C$

$= \frac{1}{2}\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{2x}\right)+C$

16. If C is the centre of the locus of P, which is a point dividing AB in the ratio 3 : 2 where A (1, 2) and B is a point on the curve $x^2 + y^2 = 16$, then the length AC is

(1) $\frac{3}{\sqrt{5}}$ (2) $\frac{4}{\sqrt{5}}$ (3) $\frac{9}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{5}}$

Ans. **(1)**

Sol.

$h = \frac{12\cos\theta + 2}{5}$, $k = \frac{12\sin\theta + 4}{5}$

$5h - 2 = 12\cos\theta$, $5k - 4 = 12\sin\theta$

locus of point P is

$(5x - 2)^2 + (5y - 4)^2 = 144$,

its centre : C (2/5, 4/5)

$AC = \sqrt{(1 - 2/5)^2 + (2 - 4/5)^2} = \sqrt{\frac{9}{25} + \frac{36}{25}} = \sqrt{\frac{45}{25}} = \frac{3}{\sqrt{5}}$

17. If f(x) is continuous function for all x and $\int_0^{t^2} (f(x) + x^2)dx = \frac{4}{3}t^3$ then the value of $f\left(\frac{\pi^2}{4}\right)$ is equal to

(1) $\pi - \frac{\pi^4}{16}$ (2) $\pi + \frac{\pi^4}{16}$ (3) $\pi - \frac{\pi^2}{8}$ (4) $\pi + \frac{\pi^2}{3}$

Ans. **(1)**

Sol. $\int_0^{t^2} (f(x) + x^2)dx = \frac{4}{3}t^3$

Diff. on both sides

$(f(t^2) + t^4)\, 2t = 4t^2$

$f(t^2) = 2t - t^4$

Put $t = \pi/2$

$f(\pi^2/4) = 2 \times \frac{\pi}{2} - \frac{\pi^4}{16}$

$= \pi - \frac{\pi^4}{16}$