MATHEMATICS

1. Find sum of all possible roots of $|x^2 - 8x + 15| - 2x + 7 = 0$

(1) $9 + \sqrt{3}$ (2) $5 + \sqrt{3}$

(3) $5 - \sqrt{3}$ (4) $4 + \sqrt{3}$

Ans. **(1)**

Sol. $|x^2 - 8x + 15| = 2x - 7$

Case-I : $x \geq 5$

$x^2 - 10x + 22 = 0$

$x = \dfrac{10 \pm \sqrt{12}}{2} = 5 \pm \sqrt{3}$

then $x = 5 + \sqrt{3}$

Case-II : $\dfrac{7}{2} \leq x \leq 5$

$x^2 - 8x + 15 = 7 - 2x$

$x^2 - 6x + 8 = 0$

$x = 4$

∴ Sum of roots $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$

2. The coefficient of x^{18} in the expansion of $\left(x^4 - \dfrac{1}{x^3} \right)^{15}$ is

(1) $^{14}C_7$ (2) $^{15}C_8$

(3) $^{15}C_6$ (4) $^{14}C_8$

Ans. **(3)**

Sol. $T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(-\dfrac{1}{x^3} \right)^r$

$T_{r+1} = {}^{15}C_r (-1)^r x^{60-7r}$

∴ $60 - 7r = 18 \Rightarrow$ $r = 6$

∴ $T_7 = {}^{15}C_6 (-1)^6 x^{18}$

$\Rightarrow$ Coefficient of x^{18} is $^{15}C_6$ or $^{15}C_9$

3. The sum of first 20 terms of series 5, 11, 19, 29, 41 …… is

(1) 3520   (2) 3510

(3) 3500   (4) 3505

Ans. **(1)**

Sol. 5 11 19 29 41….

6   8   10   12 $\rightarrow$ 1st difference is AP

$\Rightarrow$ $T_n = an^2 + bn + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$ — $3a + b = 6$

$T_3 = 9a + 3b + c = 19$ — $5a + b = 8$ — $a = 1, b = 3, c = 1$

$T_n = n^2 + 3n + 1$

$S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20}\left(n^2 + 3n + 1\right)$

$S_{20} = \dfrac{20 \times 21 \times 41}{6} + 3 \times \dfrac{20 \times 21}{2} + 20$

$S_{20} = 2870 + 630 + 20$

$S_{20} = 3520$

4. The number of ways to distribute 20 chocolates among three students such that each student gets atleast one chocolate is

(1) $^{22}C_2$ (2) $^{19}C_2$ (3) $^{19}C_3$ (4) $^{22}C_3$

Ans. **(2)**

Sol. Let x, y, z are number of chocolates three students get

$x + y + z = 20$; $x, y, z \geq 1$

$\therefore$ no. of ways is $^{19}C_2$

5. In the expansion of $(2^{1/4} + 3^{-1/4})^n$, the ratio of 5th term from start and 5th term from end is $\sqrt{6} : 1$, then find 3rd term

(1) $30\sqrt{3}$ (2) $60\sqrt{3}$ (3) 30 (4) $50\sqrt{3}$

Ans. **(2)**

Sol. $\dfrac{^nC_4 (2^{1/4})^{n-4}(3^{-1/4})^4}{^nC_4 (3^{-1/4})^{n-4}(2^{1/4})^4} = \sqrt{6}$

$\left(\dfrac{2^{1/4}}{3^{-1/4}}\right)^{n-8} = \sqrt{6}$

$(6)^{\frac{n-8}{4}} = \sqrt{6}$

$n - 8 = 2$

$n = 10$

$T_3 = {}^{10}C_2 (2^{1/4})^8 (3^{-1/4})^2$

$= {}^{10}C_2 \times (\sqrt{2})^4 \times \dfrac{1}{\sqrt{3}} = 60\sqrt{3}$

6. Let $A = [a_{ij}]_{2 \times 2}$ be a matrix and $A^2 = I$ where $a_{ij} \neq 0$. If a is sum of diagonal elements and $b = \det(A)$, then $3a^2 + 4b^2$ is

(1) 10   (2) 12   (3) 4   (4) 8

Ans. **(3)**

Sol. $A^2 = I$

So $A^2 - 0A - I = O \Rightarrow \lambda^2 - 0\lambda - 1 = 0$

Here $a = \lambda_1 + \lambda_2 = 0$

$b = \lambda_1 \lambda_2 = -1$

7. Let $I(x) = \int \dfrac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2}\, dx$. If $I(0) = 1$ then $I\left(\dfrac{\pi}{4}\right)$ is equal to

(1) $-\dfrac{\pi^2}{4\pi+16} + 2\ell n\left(\dfrac{\pi+4}{4\sqrt{2}}\right) + 1$ (2) $\dfrac{\pi^2}{4\pi+16} - 2\ell n\left(\dfrac{\pi+4}{4\sqrt{2}}\right) + 1$

(3) $-\dfrac{\pi^2}{\pi+4} + 2\ell n\left(\dfrac{\pi+1}{\sqrt{2}}\right) + 1$ (4) $\dfrac{\pi^2}{\pi+16} + 2\ell n\left(\dfrac{\pi+1}{4\sqrt{2}}\right) + 1$

Ans. **(1)**

Sol. Using integration by parts

$I(x) = x^2 \cdot \dfrac{(-1)}{x \tan x + 1} - \int 2x \cdot \dfrac{(-1)}{x \tan x + 1}\, dx$

$\qquad = -\dfrac{x^2}{x \tan x + 1} + 2\int \dfrac{x \cos x}{x \sin x + \cos x}\, dx$

$\Rightarrow I(x) = \dfrac{-x^2}{x \tan x + 1} + 2\ell n|x \sin x + \cos x| + c$

put $x = 0$

$c = 1$

$\therefore I\left(\dfrac{\pi}{4}\right) = \dfrac{\dfrac{-\pi^2}{16}}{\dfrac{\pi}{4}+1} + 2\,\ell n\left(\dfrac{\dfrac{\pi}{4}+1}{\sqrt{2}}\right) + 1$

$I\left(\dfrac{\pi}{4}\right) = -\dfrac{\pi^2}{4\pi+16} + 2\ell n\left(\dfrac{\pi+4}{4\sqrt{2}}\right) + 1$

8. If a_1, a_2, …….. a_n are in arithmetic progression with common difference d > 0, then find

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+....+\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}\right)$$

Ans. **(1)**

Sol. $\dfrac{1}{d}\sqrt{\dfrac{d}{n}}\left[\left(\sqrt{a_2}-\sqrt{a_1}\right)+\left(\sqrt{a_3}-\sqrt{a_2}\right)+....+\left(\sqrt{a_n}-\sqrt{a_{n-1}}\right)\right]$

$=\dfrac{1}{d}\sqrt{\dfrac{d}{n}}\left(\sqrt{a_n}-\sqrt{a_1}\right)$

$=\dfrac{1}{d}\sqrt{\dfrac{d}{n}}\left(a_1+nd\right)^{1/2}$

$=\dfrac{1}{d}\sqrt{\dfrac{d}{n}}\times\sqrt{n}\left(d+\dfrac{a_1}{n}\right)^{1/2}$

$=\dfrac{1}{d}\sqrt{d}\times\sqrt{d}\left(1+\dfrac{a_1}{nd}\right)^{1/2}$

$=1$

9. A pair of dice is rolled 5 times. Let getting a total of 5 in a single throw is considered as success. If probability of getting atleast four successes is $\dfrac{x}{3}$ then x is equal to

(1) $\dfrac{41}{9^5}$ (2) $\dfrac{41}{9^4}$ (3) $\dfrac{123}{9^5}$ (4) $\dfrac{123}{9^4}$

Ans. **(3)**

Sol. P(success) = $\dfrac{4}{36}=\dfrac{1}{9}$

P(atleast four success) = $^5C_4\left(\dfrac{1}{9}\right)^4\cdot\dfrac{8}{9}+\left(\dfrac{1}{9}\right)^5=\dfrac{x}{3}$

$\Rightarrow x = \dfrac{41\times3}{9^5}=\dfrac{123}{9^5}$

10. Let f(x) satisfies $5f(x)+4f\left(\dfrac{1}{x}\right)=\dfrac{1}{x}+3$, then $18\displaystyle\int_1^2 f(x)\,dx$ is

(1) $10\ell n\,3-6$ (2) $5\ell n\,2-6$ (3) $10\ell n2-6$ (4) $5\ell n\,2-3$

Ans. **(3)**

Sol. $5f(x) + 4f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 3$ .....(i)

Replace $x \to \dfrac{1}{x}$

$5f\left(\dfrac{1}{x}\right) + 4f(x) = x + 3$ .....(ii)

By (i) & (ii)

$9f(x) = \dfrac{5}{x} - 4x + 3$

$18\displaystyle\int_{1}^{2} f(x)\,dx = \int_{1}^{2} \left(\dfrac{10}{x} - 8x + 6\right) dx = 10\ell n\,2 - 6$

11. If image of point P(1, 2, 3) about the plane 2x − y + 3z = 2 is point Q, then area of ΔPQR is where R is (4, 10, 12)
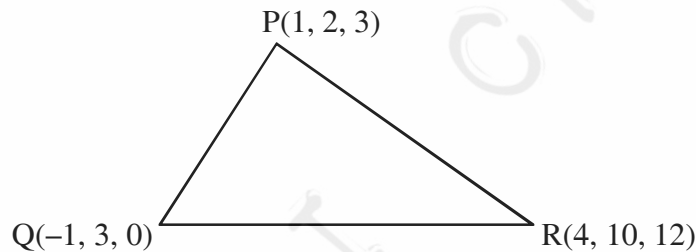
(1) $\dfrac{1}{2}\sqrt{1531}$　　　(2) $\sqrt{1531}$　　　(3) $\dfrac{1}{4}\sqrt{1531}$　　　(4) $\dfrac{1}{2}\sqrt{1351}$

Ans. **(1)**

Sol. Image formula w.r.t P

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{3} = -2\dfrac{(2\times1 - 2 + 3\times3 - 2)}{2^2 + 1^2 + 3^2}$

$\Rightarrow$　　Q(−1, 3, 0)

P(1, 2, 3)

Q(−1, 3, 0)　　　　　　　　　　R(4, 10, 12)

Area $= \dfrac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 7 & 12 \\ 2 & -1 & 3 \end{matrix}\right\|$
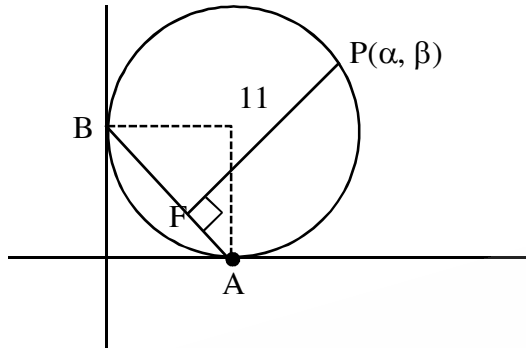
$= \dfrac{1}{2}\left|33\hat{i} + 9\hat{j} - 19\hat{k}\right|$

Area $= \dfrac{1}{2}\sqrt{(33)^2 + 9^2 + (19)^2}$

$= \dfrac{1}{2}\sqrt{1089 + 81 + 361} = \dfrac{1}{2}\sqrt{1531}$

12. Circle in Ist quadrant touches both the axes at A & B. If length of perpendicular from P(α, β) on circle to chord AB is equal to 11, Find $\alpha.\beta$

Ans. **(121)**

Sol. C : $(x - r)^2 + (y - r)^2 = r^2$; $\alpha^2 + \beta^2 - 2r(\alpha + \beta) + r^2 = 0$

$\alpha^2 + \beta^2 - 2r\left(11\sqrt{2} + r\right) + r^2 = 0$

$\alpha^2 + \beta^2 - 22\sqrt{2}\ r - r^2 = 0$

$PF = \dfrac{\alpha + \beta - r}{\sqrt{2}} = 11$

$\alpha + \beta = 11\sqrt{2} + r$

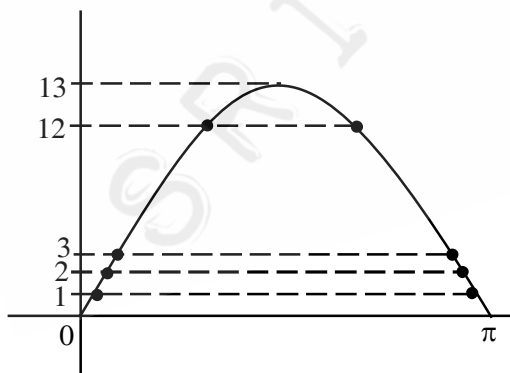$\alpha^2 + \beta^2 + 2\alpha\beta = 242 + r^2 + 22r\sqrt{2}$

$\alpha\beta = 121$

13. If f(x) = [a + 13 sinx] & x $\in$ (0, π), then number of non-differentiable points of f(x) are [where 'a' is integer]

Ans. **(25)**

Sol. Points where $\sin x = \dfrac{1}{13}, \dfrac{2}{13}, ..., \dfrac{12}{13}$ will be the points of non-derivability of f(x)

$\Rightarrow$ 24 points

And also where sin x = 1 $\Rightarrow$ 1 points

$\therefore$ 25 points of non-derivability

14. If $A(1, 1, 1)$, $B(0, \lambda, 0)$, $C(\lambda + 1, 0, 1)$, $D(2, 2, -2)$ are coplanar then $\sum (\lambda_i + 2)^2$ is equal to

(1) $\dfrac{80}{3}$　　　　　(2) $\dfrac{320}{9}$　　　　　(3) $\dfrac{160}{9}$　　　　　(4) $\dfrac{160}{3}$

Ans. **(3)**

Sol. $[\overrightarrow{AB} \ \overrightarrow{AC} \ \overrightarrow{AD}] = 0$

$\Rightarrow \lambda = 2, -\dfrac{2}{3}$

$\sum(\lambda_i + 2)^2 = 16 + \dfrac{16}{9} = \dfrac{160}{9}$

DATA FICTITIOUS

15. If $[x + 6] + [x + 3] \leq 7$ and let call its solution as set A and set B is the solution of inequality $3^{5x-8} < 3^{-3x}$

(1) $B \subset A, A \neq B$　　　(2) $A \subset B, A \neq B$　　　(3) $A \cap B = \phi$　　　(4) $A \cup B = R$

Ans. **(2)**

Sol. $2[x] \leq -2 \Rightarrow [x] \leq -1 \Rightarrow x < 0$
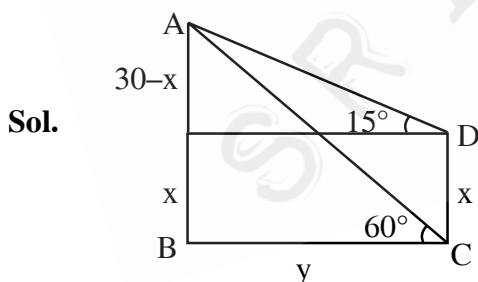
A is $(-\infty, 0)$

$5x - 8 < -3x \Rightarrow x < 1 \Rightarrow$ B is $(-\infty, 1)$

Hence $A \subset B, A \neq B$

16. Height of tower AB is 30 m where B is foot of tower. Angle of elevation from a point C on level ground to top of tower is 60° and angle of elevation of A from a point D x m above C is 15° then find area of quadrilateral ABCD.

(1) $300(\sqrt{3} - 1)$　　　(2) $600(\sqrt{3} - 1)$　　　(3) $150(\sqrt{3} - 1)$　　　(4) $100(\sqrt{3} - 1)$

Ans. **(2)**

Sol.

$\tan 60° = \dfrac{30}{y} = \sqrt{3}$

$\Rightarrow y = 10\sqrt{3}$

$$\tan 15° = \frac{30 - x}{y}$$

$$(2 - \sqrt{3})10\sqrt{3} = 30 - x$$

$$x = 30 - 20\sqrt{3} + 30$$

$$x = 60 - 20\sqrt{3}$$

Area of ABCD = xy = $(60 - 2\sqrt{3}). 10\sqrt{3}$

$$= 600(\sqrt{3} - 1)$$

17. Equivalent statement to $(p \rightarrow q) \vee (r \rightarrow q)$ will be

(1) $(p \wedge r) \rightarrow q$ (2) $(p \vee r) \rightarrow q$

(3) $(q \rightarrow r) \vee (p \vee r)$ (4) $(r \rightarrow p) \wedge (q \rightarrow r)$

Ans. **(1)**

Sol.

p	q	r	p →q	r→q	(p→q) ∨(r→q)	(p ∧ r)	(p ∧ r) → q
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	F	F	F	T	F
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	T	T	F	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	F	T

18. For two groups of 15 sizes each, mean and variance of first group is 12, 14 respectively, and second group has mean 14 and variance of σ^2. If combined variance is 13 then find variance of second group ?

(1) 9   (2) 11   (3) 10   (4) 12

Ans. **(3)**

Sol. $\overline{x} = 12, \ \sigma_1^2 = 14, \ \overline{y} = 14, \ \sigma_2^2 = \sigma^2, \ n_1 = n_2 = 15$

$$\sigma_1^2 = 14 = \frac{\sum x_i^2}{15} - (12)^2 \Rightarrow \sum x_i^2 = 2370, \sum xi = 180$$

$$\sigma_2^2 = \frac{\sum y_i^2}{15} - (14)^2, \ \sum y_i = 210$$

$$13 = \frac{\sum x_i^2 + \sum y_i^2}{30} - \left(\frac{15\overline{x} + 15\overline{y}}{30}\right)^2$$
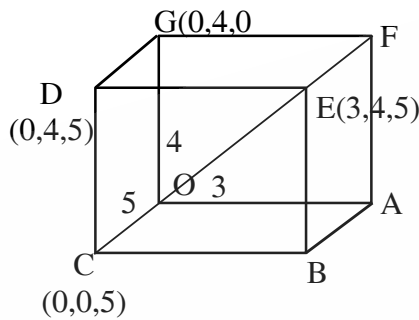
$$13 = \frac{2370 + \sum y_i^2}{30} - (13)^2$$

$$\sum y_i^2 = 3090 \Rightarrow \sigma_2^2 = \frac{3090}{15} - (14)^2 = 10$$

19. A rectangular parallelepiped with edges along x, y, z axis has length of 3, 4, 5 respectively. Find the shortest distance of the body diagonal from one of the edges parallel to z-axis which is skew to the diagonal

(1) $\dfrac{16}{5}$ (2) $\dfrac{15}{\sqrt{34}}$ (3) $\dfrac{12}{5}$ (4) $\dfrac{9}{5}$

Ans. **(3)**

Sol. Equation of diagonal OE $\vec{r} = 0 + \lambda\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$

Equation of edge GD

$\vec{r} = 4\hat{j} + \mu\hat{k}$

Shortest distance $= \left|\text{projection of } 4\hat{j} \text{ on}\left(3\hat{j} - 4\hat{i}\right)\right|$

$= \dfrac{12}{\sqrt{9+16}} = \dfrac{12}{5}$

20. If $^{2n}C_3 : {}^{n}C_3 = 10$, then $\dfrac{n^2 + 3n}{n^2 - 3n + 4}$ is equal to

Ans. **(2)**

Sol. $\dfrac{^{2n}C_3}{^{n}C_3} = 10 \Rightarrow \dfrac{2n.(2n-1).(2n-2)}{n.(n-1)(n-2)} = 10$

$\Rightarrow \dfrac{(2n-1).2}{n-2} = 5$

$\Rightarrow n = 8$

$\therefore \dfrac{n^2 + 3n}{n^2 - 3n + 4} = \dfrac{88}{44} = 2$

Sri Chaitanya IIT Academy., India

21. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is a vector perpendicular

to $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot \vec{d} = 18$ then find $|\vec{a} \times \vec{d}|^2$

(1) 720   (2) 700   (3) 360   (4) 300

Ans. **(1)**

Sol. $\vec{d} = \lambda(\vec{b} \times \vec{c}) = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a} \cdot \vec{d} = 18$

$\Rightarrow \lambda = 2$

$\therefore |\vec{a} \times \vec{d}|^2 = \vec{a}^2 \vec{d}^2 - (\vec{a} \cdot \vec{d})^2$

$\Rightarrow |\vec{a} \times \vec{d}|^2 = 29 \times 36 - 324 = 1044 - 324 = 720$
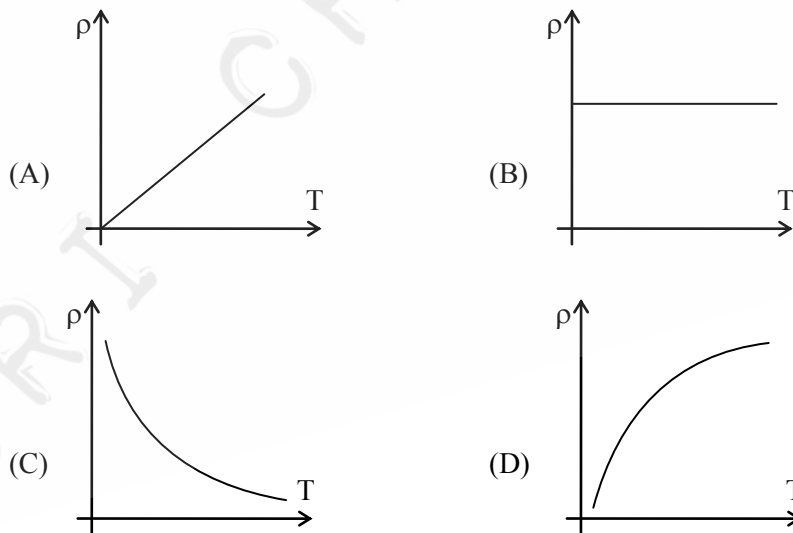
PHYSICS

1. Choose correct graph of electric potential for uniformly charged hollow sphere.
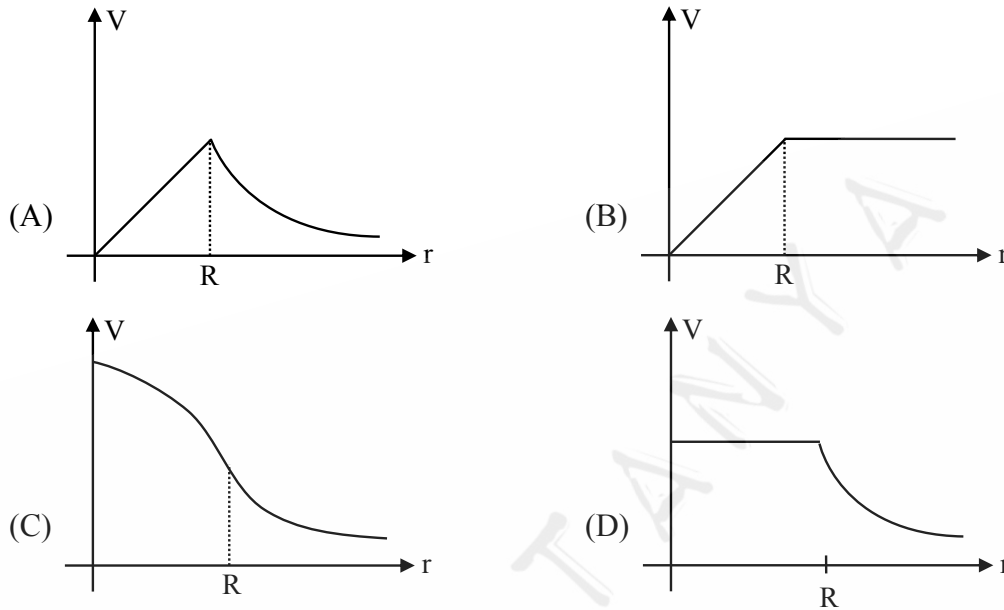
Ans. **(D)**

Sol. **Theoretical**

2. Choose correct graph of resistivity and temperature for semi-conductor material.
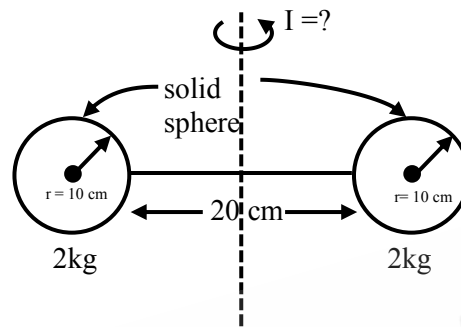

Ans. **(C)**

Sol. $\rho = \dfrac{m}{ne^2\tau}$

As T increases τ decreases but n increases but n is dominant over τ so ρ decreases with increase in temperature

3. Find moment of inertia about axis shown which is equidistant from both spheres

Ans. $\dfrac{88}{500}(kg - m^2)$

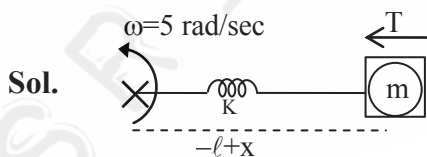**Sol.** $I = \left[\dfrac{2}{5}Mr^2 + M(0.2)^2\right] \times 2$

$= \left[\dfrac{2}{5} \times 2 \times (0.1)^2 + 2 \times (0.2)^2\right] \times 2$

$= \left[\dfrac{4}{500} + \dfrac{8}{100}\right] \times 2$

$= \dfrac{44 \times 2}{500} = \dfrac{88}{500}(kg - m^2)$

4. A block of mass 100 g is attached with spring of natural length 20 cm and force constant 7.5 N/m. Now system is rotated with constant angular velocity 5 radian/s on horizontal plane. Find out tension in the spring?

Ans. 0.75 N

Sol.

ℓ = 20 cm (natural length of spring)

m = given 100 gm

k = 7.5 N/m

$mw^2(\ell + x)$

$Kx = mw^2 (\ell + x)$

$$x = \frac{m\omega^2\ell}{K - m\omega^2}$$

$$T = Kx = \left| \frac{m\omega^2\ell K}{K - m\omega^2} \right|$$

$$T = \frac{0.1 \times 25 \times 0.2 \times 7.5}{7.5 - 0.1 \times 25}$$

$$T = \frac{7.5 \times 0.1 \times 5}{5}$$

T = 0.75 N

5.   **Assertion :** Range is maximum at θ = 45º.

Reason : Range is maximum when sin (2θ) = 1.

(A) Both assertion and reason are true & reason is the correct explanation of assertion.

(B) Both assertion and reason are true but reason is not correct explanation of assertion.

(C) Assertion is true and reason is false.

(D) Assertion is false and reason is false.

Ans.   (A)

Sol. $R = \dfrac{u^2 \sin(2\theta)}{g}$

For R_max   sin (2θ) = 1

2θ = 90º

θ = 45º

6.   **Assertion :** The Moon doesn't have atmosphere.

Reason : Escape velocity of the Moon is less than that of the Earth.

(A) Both assertion and reason are true & reason is the correct explanation of assertion.

(B) Both assertion and reason are true but reason is not correct explanation of assertion.

(C) Assertion is true and reason is false.

(D) Assertion is false and reason is true.

Ans.   (A)

Sol. $V_{esc} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G}{R} \times \dfrac{\rho 4}{3}\pi R^3}$

$V_{esc} \propto R$.

Since moon's radius is very small compared to earth.

V_{esc} at moon is quite low and gas molecules attains escape velocity at normal temperature on moon.

7. For below transition of e^{-1} of H-atom find out shortest wavelength out of given transition
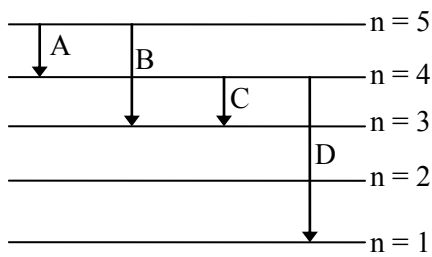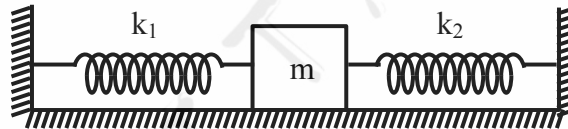

    (A) A   (B) B   (C) C   (D) D

Ans. **(D)**

Sol. Energy from n = 4 to 1 is maximum so wavelength is minimum.

8. Find period of oscillation if mass 'm' is displaced parallel to earth's surface & released?

  (A) $2\pi\sqrt{\dfrac{k_1 + k_2}{m}}$ (B) $2\pi\sqrt{\dfrac{k_1 k_2}{m(k_1 + k_2)}}$ (C) $2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$ (D) $2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

Ans. **(C)**

Sol. $k_{eq} = k_1 + k_2$ (since springs are parallel)

$$T = 2\pi\sqrt{\dfrac{m}{k_{eq}}} = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$$

9. A planet has density same as that of Earth and mass is twice that of Earth. If the weight of an object on Earth is "W" then the weight on the planet is :

    (A) $2^{\frac{2}{3}} W$ (B) $2^{\frac{1}{3}} W$ (C) $2^{\frac{4}{3}} W$ (D) W

Ans. **(B)**

Sol. Planet with mass M has radius R

Planet with mass 2M has radius R'

$$\rho\, \frac{4}{3}\pi R^3 = M$$
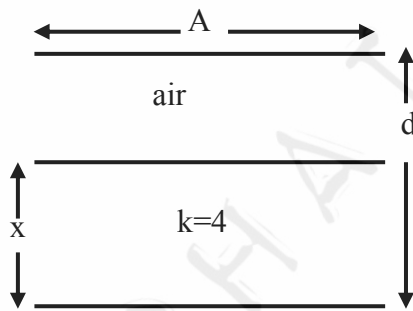
$$\rho \, \frac{4}{3} \pi R'^3 = 2M$$

$$\Rightarrow R' = 2^{\frac{1}{3}} R$$

$$= 2 \frac{GM}{2^{\frac{2}{3}} R^2} = 2^{\frac{1}{3}} \frac{GM}{R^2} = 2^{\frac{1}{3}} W$$

$$\therefore W' = 2^{\frac{1}{3}} W$$

10. When $x = \dfrac{d}{3}$ then capacitance is $C_1 = 2\mu f$

then if $x = \dfrac{2d}{3}$ then capacitance C_2 will be (in μC)

Ans. (3)

Sol. $\dfrac{1}{C_{eq}} = \dfrac{1}{C_{air}} + \dfrac{1}{C_{di}}$

$$\frac{1}{C_{eq}} = \frac{2d}{3(\in_0 A)} + \frac{d}{(3)4 \in_0 A} = \frac{3d}{4A \in_0}$$

$$C_{eq} = \frac{4A \in_0}{3d} = 2\mu F$$

$$\frac{A \in_0}{d} = 1.5\mu F$$

$$\frac{1}{C'_{eq}} = \frac{d}{3(\in_0 A)} + \frac{2d}{(3)4 \in_0 A} = \frac{(4+2)d}{12 \in_0 A} = \frac{6d}{12 \in_0 A} \Rightarrow C'_{eq} = 2\left[\frac{\in_0 A}{d}\right] = 3\mu F$$

11. In which condition EMF will be induced in loop.

Situation-S1 : A loop is moving with uniform velocity in a uniform magnetic field perpendicular to its plane.

Situation-S2 : A loop is moving with non-uniform velocity in a uniform magnetic field perpendicular to its plane.

Situation-S3 : A loop is rotating about its diameter in a uniform magnetic field.

Situation-S4 : Area of loop is changing in a uniform magnetic field.

(A) S2, S3

(B) S1, S3

(C) S2, S4

(D) S3, S4

Ans. **(D)**

Sol. S1, S2 → Flux remains constant

S3 → ϕ = BA cos θ [θ is changing]

S4 → Area is changing → EMF induces

12. If height of antenna is increased by 21%. Find % rise in its range ?

Ans. **10%**

Sol. $d = \sqrt{2Rh}$, $d \propto \sqrt{h}$

$d' \propto \sqrt{1.21\ h}$ = 1.1 h

%tage change in d = 10%

13. E = E_0 sin (ωt – kx)

B = B_0 sin (ωt – kx),

then ratio of energy density of electric field to magnetic field is :

(A) 1 : 1   (B) 1 : 3   (C) 1 : 2   (D) 2 : 1

Ans. **(A)**

Sol. In EM wave,

Energy density)$_{\text{magnetic field}}$ = Energy density)$_{\text{electric field}}$

14. A uniform current carrying cylindrical wire carries current I having radius 'a'. Magnetic field with distance x varies with relation given :

(A) $B \propto x$ for $x < a$

       $B \propto \dfrac{1}{x}$ for $x > a$

(B) $B \propto x$ for $x < a$

       B is constant   for $x > a$

(C)   B is constant   for $x < a$

       $B \propto \dfrac{1}{x}$ for $x > a$

(D) $B \propto \dfrac{1}{x}$ for $x < a$

       $B \propto x$ for $x > a$

Ans.   **(A)**

15. An electron, proton and α-particle are moving with kinetic energy 4K, K and 2K respectively. Relation between De-Broglie wavelength is :

(A) $\lambda_p > \lambda_e > \lambda_\alpha$ (B) $\lambda_p > \lambda_\alpha > \lambda_e$ (C) $\lambda e > \lambda_p > \lambda_\alpha$ (D) $\lambda_\alpha > \lambda_e > \lambda_p$

Ans.   **(C)**

Sol. $\lambda_e = \dfrac{h}{\sqrt{\dfrac{2 \times m}{2000} \times 4K}} = \dfrac{10\sqrt{20}h}{2\sqrt{2mK}}$

$\lambda_p = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2mK}}$

$\lambda_\alpha = \dfrac{h}{\sqrt{2 \times 4m \times 2K}} = \dfrac{h}{4\sqrt{mK}}$

$\lambda_e > \lambda_p > \lambda_\alpha$

16. $R_1 = (15 \pm 0.5) \; \Omega$

$R_2 = (10 \pm 0.5) \; \Omega$

Find % error in equivalent resistance if resistors R_1 and R_2 are connected in parallel ?

(A) 5%   (B) 4.33 %   (C) 3%   (D) 3.33 %

Ans.   **(B)**

Sol. $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ & $R_{eq} = \dfrac{150}{25} = 6\,\Omega$

$$\dfrac{dR_{eq}}{R_{eq}^2} = \dfrac{dR_1}{R_1^2} + \dfrac{dR_2}{R_2^2}$$

$$\dfrac{dR_{eq}}{R_{eq}} = R_{eq}\left[\dfrac{dR_1}{R_1^2} + \dfrac{dR_2}{R_2^2}\right]$$

$$= 6\left[\dfrac{0.5}{15^2} + \dfrac{0.5}{10^2}\right]$$

$\therefore \quad \% \dfrac{dR_{eq}}{R_{eq}} = 6\times0.5\times100\left[\dfrac{1}{225} + \dfrac{1}{100}\right] = 6\times0.5\times100\times\dfrac{325}{225\times100} = 4.33\%$

17. Two identical current carrying coils with same centre are placed with their planes perpendicular to each other. If i = $\sqrt{2}$A and radius of coil R = 1 m, then magnetic field at centre C is equal to :

(A) μ_0 (B) $\dfrac{\mu_0}{2}$ (C) $2\mu_0$ (D) $\sqrt{2}\ \mu_0$

Ans. **(A)**

Sol. $B_0 = \sqrt{B_1^2 + B_2^2}$

$B_0 = \sqrt{2}B$

$B_0 = \sqrt{2}\dfrac{\mu_0 I}{2R} = \sqrt{2}\dfrac{\mu_0 \times \sqrt{2}}{2\times1} = \mu_0$

18. If the length of a conductor is increased by 20% and cross-sectional area is decreased by 4%, find the percentage change in the resistance of the conductor.

Ans. 25

Sol. $R = \dfrac{\rho\ell}{A}$

$R' = \dfrac{\rho 1.2\ell}{0.96A} = \dfrac{1.25\rho\ell}{A} = 1.25\,R$

19. A car is moving with speed of 15 m/sec towards a stationary wall. A person in the car press the horn and experience the change in frequency of 40 Hz due to reflection from stationary wall. Find the frequency of horn. (Use $V_{sound\ wave}$ = 330 m/sec)

Ans. **420 Hz**

Sol. $f = f_0 \left(\dfrac{V + V_C}{V - V_C} \right)$

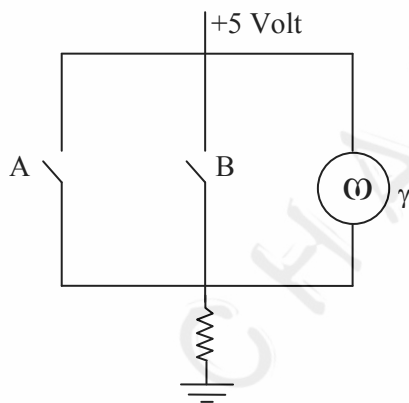$f = f_0 \left(\dfrac{330 + 15}{330 - 15} \right)$

$f = f_0 \dfrac{345}{315}$

$f - f_0 = 40$

$f_0 \left(\dfrac{345 - 315}{315} \right) = 40$

$f_0 = \dfrac{40 \times 315}{30}$

$f_0 = 420 \text{ Hz}$

20. Which logic gate is specified by given circuit

(A) AND   (B) OR   (C) NAND   (D) NOR

Ans. (D)

Sol.

A	B	γ
0	0	1
1	0	0
0	1	0
1	1	0

21. A block of mass 100 gm is placed on a smooth surface and is moving with acceleration of $a = 2x$. If change in kinetic energy can be given as $\left(\dfrac{x^n}{10} \right)$, find the value of n :

Ans. (2)

Sol. $V \dfrac{dv}{dx} = 2x \qquad \Rightarrow \qquad \therefore \qquad \displaystyle\int_0^v v\,dv = 2\int_0^x x\,dx$

$$\therefore \qquad \dfrac{v^2}{2} = x^2$$

$$\therefore \qquad \dfrac{1}{2}mv^2 = mx^2$$

$$= (0.1)\,x^2 = \dfrac{x^2}{10}$$

$$n = 2$$

22. A sphere of density ρ & mass m is moving down with constant velocity in viscous liquid of density ρ_0 find out viscous force on sphere

(A) $mg\left(\dfrac{1-\rho_0}{\rho}\right)$ (B) $2mg\left(\dfrac{1-\rho_0}{\rho}\right)$ (C) $mg\left(\dfrac{1-3\rho_0}{\rho}\right)$ (D) $mg\left(\dfrac{1+\rho_0}{\rho}\right)$

Ans. (A)

Sol. $F = mg - F_b$

$F = mg - \rho_0\dfrac{mg}{\rho}$

$F = mg\left(\dfrac{1-\rho_0}{\rho}\right)$

23. If light incident in air at an angle of incidence 45° and refracted in other medium at an angle of refraction 30°. If wavelength in air is λ_1 and frequency is ν_1 and in medium wavelength is λ_2 and frequency is ν_2 then correct possible options are :

(A) $\lambda_1 = \dfrac{\lambda_2}{\sqrt{2}}, \nu_1 = \nu_2$ (B) $\lambda_1 = \lambda_2, \nu_1 = \dfrac{\nu_2}{\sqrt{2}}$ (C) $\lambda_1 = \lambda_2, \nu_2 = \nu_1$ (D) $\lambda_2 = \dfrac{\lambda_1}{\sqrt{2}}, \nu_2 = \nu_1$

Ans. (D)

Sol. $1\sin 45° = \mu \sin 30°$

$\mu = \sqrt{2}$

$\lambda_2 = \dfrac{\lambda_{air}}{\mu} = \dfrac{\lambda_1}{\sqrt{2}}$

and frequency does not change with medium.

24. A wire of length 2 m, radius of cross-section 20 mm and Young's Modulus 2×10^{11} N/m is subjected to a force of 62.8 kN. The change in length of the wire is $p \times 10^{-5}$ (in m). Find P.

Ans. (50)

Sol. $\Delta L = \dfrac{FL}{AY} = \dfrac{62.8 \times 1000 \times 2}{3.14 \times 20 \times 20 \times 10^{-6} \times 2 \times 10^{11}} = 50 \times 10^{-5}$ m

25. In a thermodynamic process work done by gas is 1000 J & heat supplied is 200 J. Find change in internal energy of gas?

(A) 800 J   (B) −800 J   (C) 1200 J   (D) −1200 J

Ans. (B)

Sol. $Q = \Delta U + \Omega$

$\therefore 200 = \Delta U + 1000$

$\therefore \Delta U = -800$ J

26. A particle moves in a circular path with uniform speed v. When it turns by 90º, find ratio of $\dfrac{v}{|<\vec{v}>|}$?

(A) $\dfrac{\pi}{\sqrt{2}}$ (B) $\dfrac{2\pi}{\sqrt{2}}$ (C) $\dfrac{\pi}{2}$ (D) 2π

Ans. (A)

Sol. $|<\dot{v}>| = \dfrac{|\,\text{displacement}\,|}{\text{time}} = \dfrac{\sqrt{2}R}{(\pi R / v)} = \dfrac{\sqrt{2}v}{\pi}$

$\therefore \quad \dfrac{v}{|<\vec{v}>|} = \dfrac{v}{\left(\dfrac{\sqrt{2}v}{\pi}\right)} = \dfrac{\pi}{\sqrt{2}}$

27. Find out 5th orbit radius (in pm) of Li^{++} if ground orbit radius of H-atom is 51 pm.

Ans. (425)

Sol. $r_0 = 51$ pm

$r = r_0 \times \dfrac{n^2}{Z}$

$r = 51 \text{ pm} \times \dfrac{5^2}{3} = 425$

CHEMISTRY

1. Predict expression for α in terms of K_{eq} and concentration C :

$$A_2B_3 \text{ (aq)} \rightleftharpoons 2A^{3+}\text{(aq)} + 3B^{2-}\text{ (aq)}$$
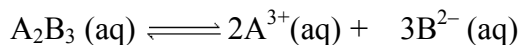
(1*) $\left(\dfrac{K_{eq}}{108C^4}\right)^{1/5}$ 　　　(2) $\left(\dfrac{K_{eq}}{5C^4}\right)^{1/5}$ 　　　(3) $\left(\dfrac{4K_{eq}}{5C^4}\right)^{1/5}$ 　　　(4) $\left(\dfrac{9K_{eq}}{5C^4}\right)^{1/5}$

Sol. $\quad A_2B_3 \text{ (aq)} \rightleftharpoons 2A^{3+}\text{(aq)} + 3B^{2-}\text{ (aq)}$

　　　C
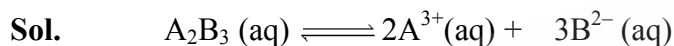
　　　$C(1-\alpha)$ 　　　$2C\alpha$ 　　　$3C\alpha$

$K_{eq} = \dfrac{(2C\alpha)^2(3C\alpha)^3}{C}$

$K_{eq} = 108C^4\alpha^5$

$\alpha = \left(\dfrac{K_{eq}}{108C^4}\right)^{1/5}$

2. Radius of first orbit of hydrogen atom is 51 pm. Determine the radius of 5th orbit of Li^{2+}

Ans. 425 pm

Sol. $\quad r_H = 51$ pm

$(r_H{}^{2+})_5 = (r_H)_1 \times \dfrac{n^2}{Z} = 51 \times \dfrac{5^2}{3} = 425$ pm

3. How many moles of $Ba_3(PO_4)_2$ will be formed by the reaction of 5 moles of $BaCl_2$ and 3 moles of $Na_3(PO_4)$.

Ans. $\dfrac{5}{3}$

Sol. $\quad 3\,BaCl_2 + 2Na_3\,PO_4 \longrightarrow Ba_3(PO_4)_2 + 6NaCl$

　　　5 mole 　　　3 mole

Moles of $Ba_3(PO_4)_2 = \dfrac{5}{3}$

4. In which of the following pairs of elements electron gain enthalpy difference is highest ?

(1) Cl, Ar   (2) Cl, Ne   (3) F, Ar   (4) F, Ne

Ans. **(2)**

Sol. Chlorine has most negative ΔH_{eg} (–349 kJ/mole) whereas Neon has most positive ΔH_{eg} (116 kJ/mole)

5. In an ionic solid element Y crystallises in ccp lattice and element X occupy $\frac{1}{3}^{rd}$ of tetrahedral void.

Find formula of ionic solid.

Ans. X_2Y_3

Sol. For 1 unit cell,

     No. of particles

     X $\frac{1}{3} \times 8$

     Y   4

∴   Formula of Ionic solid = $X_{8/3}Y_4 = X_2Y_3$

6. The value of $\log_{10}K$ for a reaction $A \rightleftharpoons B$ is

(Given $\Delta H^{\circ}_{298K} = -54.67$ kJmol^{-1}

         $\Delta S^{\circ}_{298K} = 10$ kJmol^{-1}

and $R = 8.314$ JK^{-1}mol^{-1}

$2.303 \times 8.314 \times 298 = 5705$)

Ans. **10**

Sol. $\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$

     $= -54.07 \times 1000 - 298 \times 10$

     $= -57050$

$\Delta G^{\circ} = -2.303 \ RT\log_{10}K$

$\log K = 10$

7. Determine the amount of urea (NH_2CONH_2) to be added in 1000 g of water to decrease its vapour presssure by 25%.

Sol. $\dfrac{P^{\circ} - P_S}{P^{\circ}} = \dfrac{n}{N+n} = \dfrac{1}{4}$

$\Rightarrow \ 4n = N + n$

$$n = \frac{N}{3} = \left(\frac{1000}{18}\right) \times \frac{1}{3}$$

∴ Amount of urea is $\frac{(1000)}{18 \times 3} \times 60 = \frac{10000}{9}$ gm

≈ 1111.1 gram

8. Which of the following slows down the process of setting of the cement ?

Ans. **Gypsum**

9. Number of ambidentate ligands in given complex $[M(en)(SCN)_4]$:

Ans. **4**

Sol. SCN^- is an ambidentate ligand S & N both are donor atom.

10. $2[Au(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2Au\downarrow$
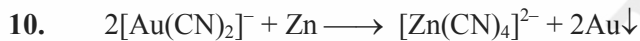
(A) Redox reaction   (C) Displacement reaction

(B) Combination reaction   (D) Decomposition reaction

(1*) A & B   (2) B only   (3) A & D   (4) B & D

Sol. $2[Au(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2Au\downarrow$
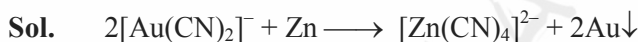
It is a redox, displacement reaction.

11. $A \Rightarrow$ Spin only magnetic moment of $[Fe(CN)_6]^{-3}$ is 1.73 B.M. and $[Fe(H_2O)_6]^{+3}$ is 5.92 B.M.

$R \Rightarrow$ In both cases Fe have +3 oxidation state

Ans. Both A & R are correct but R is not the correct explanation

Sol. $[Fe(CN)_6]^{-3}$: Fe^{+3} : $3d^5$ with S.F.L

$$\Rightarrow n = 1$$

Magnetic moment = 1.73 B.M

$[Fe(H_2O)_6]^{+3}$ Fe^{+3} : $3d^5$ with W.F.L

$$\Rightarrow n = 5$$

Magnetic moment = 5.92 B.M

12. Assertion: Radius of H^+ is 1.5×10^{-3} pm

Reason: H^+ cannot exist independently

Sol. Both assertion and reason are correct but reason is not a correct explanation of assertion.

13. Oxidation number of Mo in Ammonophosphomolybdate

Ans. **6**

Sol. $(NH_4)_3PMo_{12}O_{40}$ or $(NH_4)_3PO_4.12MoO_3$

$+3 + 5 +12x - 80 = 0$

$12x = 80 - 8$

$12x = 72$

$x = 6$

14. Which of following are reducing and oxidising agent respectively.

(1) Eu^{+2}, Ce^{+4} (2) Ce^{+3}, Ce^{+4}

(3) Eu^{+4}, Eu^{+2} (4) Tb^{+2}, Ce^{2+}
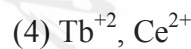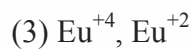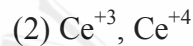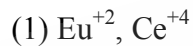
Ans. **(1)**

Sol. $Eu^{2+} \longrightarrow Eu^{3+} + e^-$

$Eu^{2+} \longrightarrow$ Good reducing agent

$e^- + Ce^{4+} \longrightarrow Ce^{3+}$

Ce^{4+} is a good oxidising agent

15.

Column-I	Column-II
(P) N_2O_5	(i) N–N bond
(Q) N_2O	(ii) N–O–N bond
(R) N_2O_4	(iii) N=N / N≡N bond
(S) NO_2	(iv) N=O bond

Ans. P – (ii), Q – (iii), R – (i), S – (iv)

Sol.
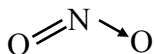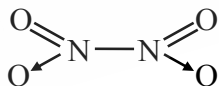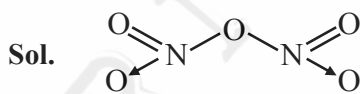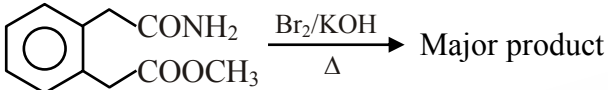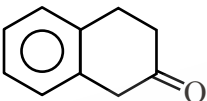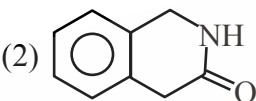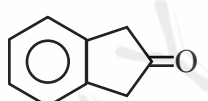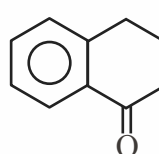
16. Polymer which is named as orlon

(1) Polyamide

(2) Polyacrylonitrile

(3) Polycarbamate

(4) Polyethene

Ans. **(2)**

17.

$\xrightarrow[\Delta]{Br_2/KOH}$ Major product

(1) (2) (3) (4)

Ans. **(2)**

Sol.

18.
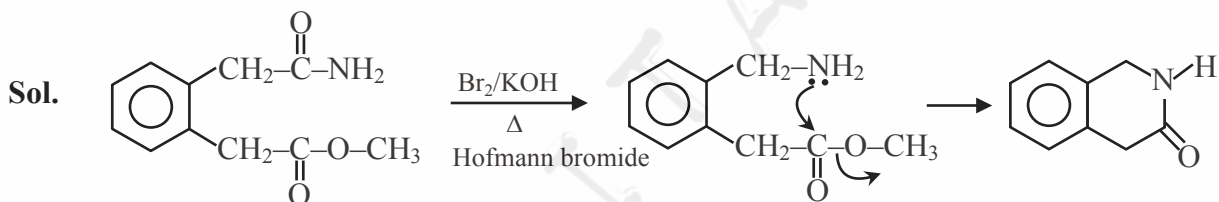
Column I	Column II
(i) Vitamin A	(a) Beri-beri
(ii) Vitamin C (Ascorbic acid)	(b) Cheilosis
(iii) Riboflavin	(c) Xerophthalmia
(iv) Thiamine	(d) Scurvy

(1) i→c, ii→d, iii→a, iv→b

(2) i→c, ii→d, iii→b, iv→a

(3) i→d, ii→c, iii→b, iv→a

(4) i→c, ii→b, iii→d, iv→a

Ans. **(2)**

19. Photochemical smog found mainly in

(1) Industrial area

(2) Marshy place
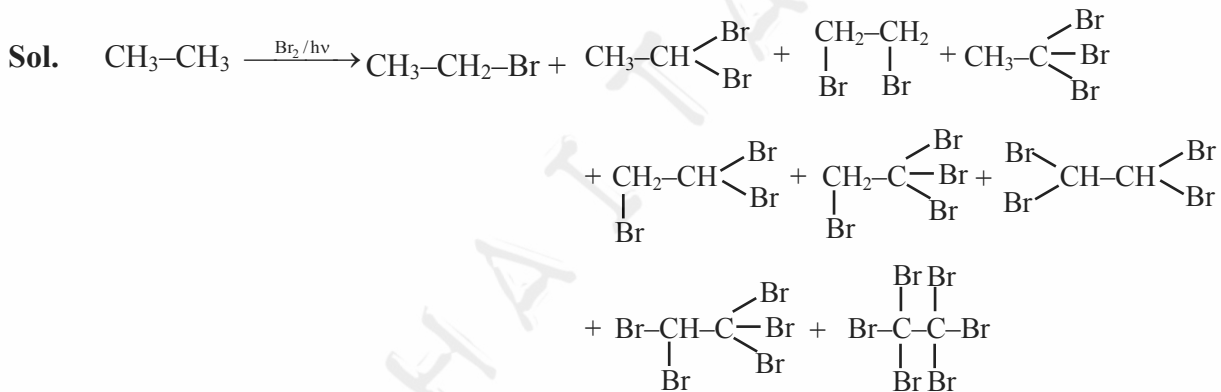
(3) Hilly area of Himachal

(4) Cold humid climate

Ans. **(1)**

20.   **Column I (Chemical reactions)**   **Column II (Enzymes used)**

   (i) Glucose $\rightarrow CO_2$ + Ethanol   (a) Pepsin

   (ii) Sucrose $\rightarrow$ Glucose + Fructose   (b) Diastase

   (iii) Starch $\rightarrow$ Maltose   (c) Zymase

   (iv) Protein $\rightarrow$ Amino acids   (d) Invertase

   (1) i$\rightarrow$c, ii$\rightarrow$d, iii$\rightarrow$b, iv$\rightarrow$a   (2) i$\rightarrow$d, ii$\rightarrow$c, iii$\rightarrow$b, iv$\rightarrow$a

   (3) i$\rightarrow$c, ii$\rightarrow$d, iii$\rightarrow$a, iv$\rightarrow$b   (4) i$\rightarrow$c, ii$\rightarrow$b, iii$\rightarrow$d, iv$\rightarrow$a

Ans.   **(1)**

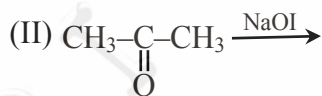**21.**   How many bromo products are formed when ethane is reacted with excess of Br_2 on heating?

Ans.   **(9)**

Sol. $CH_3-CH_3 \xrightarrow{Br_2/h\nu} CH_3-CH_2-Br$ + $CH_3-CH\begin{smallmatrix}Br\\Br\end{smallmatrix}$ + $\underset{\underset{Br}{|}\,\underset{Br}{|}}{CH_2-CH_2}$ + $CH_3-C\begin{smallmatrix}Br\\Br\\Br\end{smallmatrix}$

   + $\underset{\underset{Br}{|}}{CH_2-CH}\begin{smallmatrix}Br\\Br\end{smallmatrix}$ + $\underset{\underset{Br}{|}}{CH_2-C}\begin{smallmatrix}Br\\Br\\Br\end{smallmatrix}$ + $\begin{smallmatrix}Br\\Br\end{smallmatrix}CH-CH\begin{smallmatrix}Br\\Br\end{smallmatrix}$

   + $Br-\underset{\underset{Br}{|}}{CH}-C\begin{smallmatrix}Br\\Br\end{smallmatrix}$ + $Br-\underset{\underset{Br}{|}}{\overset{\overset{Br}{|}}{C}}-\underset{\underset{Br}{|}}{\overset{\overset{Br}{|}}{C}}-Br$

22.   Match the following with the correct name of reaction

   (I) $CH_3-COOH \xrightarrow{Red-P + Br_2}$ (P) Gattermann Koch reaction

   (II) $CH_3-\underset{\underset{O}{||}}{C}-CH_3 \xrightarrow{NaOI}$ (Q) Hell Volhard Zelinsky
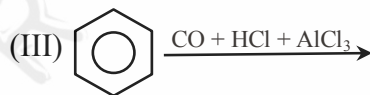
   (III) [benzene ring] $\xrightarrow{CO + HCl + AlCl_3}$ (R) Iodoform reaction
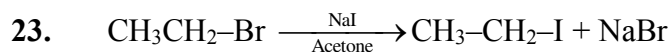
   (1) (I) $\rightarrow$ (Q), (II) $\rightarrow$ (R), (III) $\rightarrow$ (P)

   (2) (I) $\rightarrow$ (R), (II) $\rightarrow$ (Q), (III) $\rightarrow$ (P)

   (3) (I) $\rightarrow$ (Q), (II) $\rightarrow$ (P), (III) $\rightarrow$ (R)

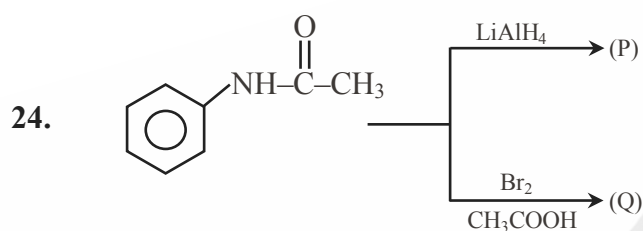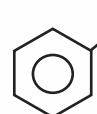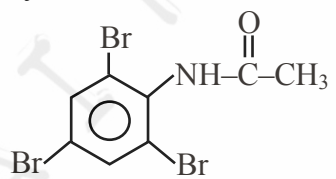   (4) (I) $\rightarrow$ (P), (II) $\rightarrow$ (Q), (III) $\rightarrow$ (R)
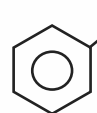
Ans.   **(1)**

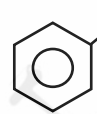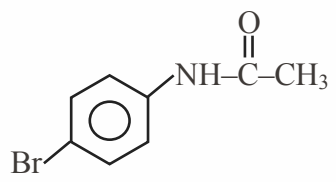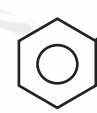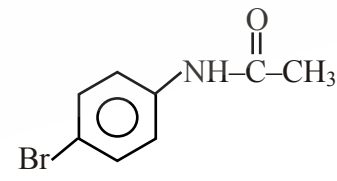**23.** $CH_3CH_2-Br \xrightarrow[\text{Acetone}]{\text{NaI}} CH_3-CH_2-I + NaBr$
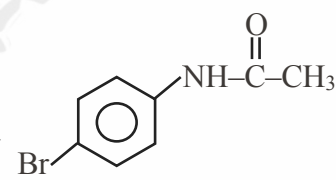
Which of the following statement is correct ?

(1) Acetic acid solvent can take in above reaction.

(2) NaI is soluble in acetone but NaBr is precipitate in acetone

(3) NaI is precipitated in acetone but NaBr is soluble in acetone

(4) When acetone is taken in solvent transition state is highly polar

Ans. **(2)**

24.

Product (P) and (Q) are respectively

(1) and

(2) and

(3) and

(4) and

Ans. **(3)**