PART : MATHEMATICS

1. If mean and variance of series x, y, 12, 6, 4, 8, 12, 10 are 9 and 9.25 respectively (x > y) then the value of 3x – 2y is

Ans. **(25)**

Sol. $\frac{x+y+12+6+4+8+12+10}{8} = 9 \Rightarrow x + y = 20$ ______ (1)

$\sigma^2 + \bar{x}^2 = \frac{1}{n}\sum x_i^2$

$\Rightarrow \quad 9.25 + 9^2 = \frac{1}{8}\ (x^2 + y^2 + 12^2 + 6^2 + 4^2 + 8^2 + 12^2 + 10^2)$

$74 + 8 \times 81 = x^2 + y^2 + 504$

$218 = x^2 + y^2$ __________(2)

(1), (2) $\Rightarrow x^2 + (20 - x)^2 = 218$

$\Rightarrow x^2 - 20x + 91 = 0$

$x = 7, 13$

$\therefore \quad x = 13\ ,\ y = 7$

$3x - 2y = 39 - 14 = 25$

2. If $P = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = P^T AP$ then $PQ^{2007} P^T =$

(1) $\begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 2007 & 1 \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 2007 \\ 1 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 1 \\ 1 & 2007 \end{bmatrix}$

Ans. **(1)**

Sol. $PQ^{2007} P^T = P(P^T AP)\ (P^T AP)\ (P^T AP)$ ------ $(P^T AP)\ P^T$

$= IAIA$ ----- IAI

$= A^{2007}$

$= \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$PP^T = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

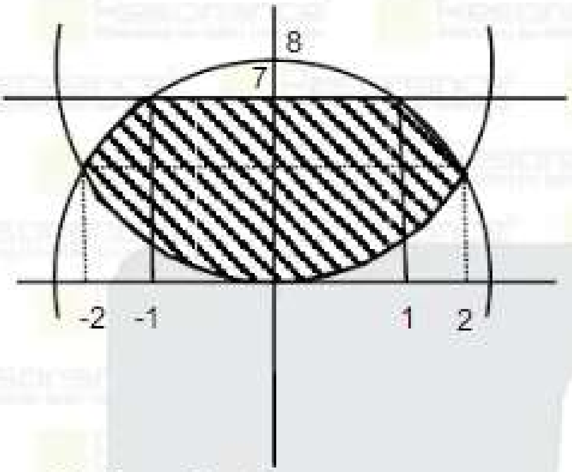
$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

------ $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

3. Area under the curves $y \le 7$, $x^2 \le y \le 8 - x^2$ is

Ans. **20**



$y \le 7$, $x^2 \le y \le 8 - x^2$

$x^2 = y$ & $y = 8 - x^2$

Point of intersection

$x^2 = 8 - x^2$

$2x^2 = 8$

$x = \pm 2$

$$A = 2\int_0^1 \left(7 - x^2\right)dx + 2\int_1^2 \left(8 \quad x^2 - x^2\right)dx$$

$$= 2\left[7x - \frac{x^2 3}{3}\right]_0^1 + 2\left[8x - \frac{2x^3}{3}\right]_1^2$$

$$= 2\left(7 - \frac{1}{3} + 8(2-1) - \frac{2}{3}(8-1)\right)$$

$$= \left(7 - \frac{1}{3} + 8 - \frac{14}{3}\right)$$

$= 2\ (15 - 5)$

$= 20$

4. Negative of $(P \to q) \to (q \to p)$ is

(1) $\sim p \vee q$ (2) $\sim p \wedge q$ (3) $p \wedge q$ (4) $p \vee q$

Ans. (2)

Sol.

P	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$	$\sim((p \to q) \to (q \to p))$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

5. The highest integer n $\in$ N for which 66! is divisible by 3^n is

(1) 21 (2) 30 (3) 31 (4) 39

Ans. (3)

Sol. Method –I By formula [.]

Exponent of 3 in 66! is $\left[\frac{66}{3}\right]+\left[\frac{66}{9}\right]+\left[\frac{66}{27}\right]+\left[\frac{66}{81}\right]$

= 22 +7 + 2 + 0 = 31

66! = 3^{31} (Some integer)

Ans 31.

6. If A is a 3×3 matrix with |A| = 2 such that |adj (adj(adj(2A)))| = 4^n then the value of n is

(1) 4 (2) 8 (3) 16 (4) 32

Ans. (3)

Sol. |adj (adj(adj(2A)))| = 4^n

$$\left(\left(|2A|^2\right)^2\right)^2 = 4^n$$

$$(|2A|)^8 = 4^n$$

$$(2^3|A|)^8 = 2^{2n}$$

$$(2^4)^8 = 2^{2n}$$

$2^{32} = 2^{2n} \Rightarrow 2n = 32$

n =16

7. Number of permutation of word " INDEPENDENCE " when all vowels are together

(1) 13540 (2) 16800 (3) 12540 (4) 12340

Ans. (2)

Sol. Vol. I, , E, E, E, E

Constant N, D, P, N, D, N, C

No of ways = $\frac{8!}{3!2!}.\frac{5!}{4!}$ = 16800

8. In how many ways 7 boys and 5 girls can sit round a table such that no two girls are together.

(1) 1814400 (2) 1814000 (3) 1800400 (4) 1800300

Ans. (1)

Sol. 7 boys can sit = 6!

which create 7 gap between then in which 5 girls have to set

No of ways = 6!. × ${}^7c_5$ 5!

= 720 × 21×120

= 1814400

9. Let $f(x) = x + \int_0^1 t(x+t)f(t)dt$ then value of $\frac{23}{2}f(0)$ is

Ans. (9)

Sol. $f(x) = x + x\int_0^1 tf(t)dt + \int_0^1 t^2 f(t)dt$

Let $\int_0^1 t\,f(t)dt = A$, $\int_0^1 t^2 f(t)dt = B$

$f(x) = x + Ax + B$
$f(t) = B + t + At$

$A = \int_0^1 t\,(B + t + At)dt$

$A = B\left(\frac{t^2}{2}\right)_0^1 + (1+A)\left(\frac{t^3}{3}\right)_0^1 \quad \Rightarrow \quad A = \frac{B}{2} + \frac{1}{3}(1+A)$

$\Rightarrow 6A = 3B + 2 + 2A$
$\Rightarrow 4A - 3B - 2 = 0$........(1)

$B = \int_0^1 t^2(B + t + At)dt \quad = \quad B = \left(\frac{t^3}{3}\right)_0^1 + (1+A)\left(\frac{t^4}{4}\right)_0^1$

$B = \frac{B}{3} + \frac{1+A}{4} \Rightarrow 12B = 4B + 3 + 3A \Rightarrow 3A - 8B + 3 = 0$ ......(2)

$F(0) = B = \frac{18}{23}$

$\frac{23}{2}f(0) = 9$

$12A - 9B - 6 = 0$
$12A - 32B + 12 = 0$
$- \quad + \quad -$

$23B - 18 = 0$

$B = \frac{18}{23}$

10. Find coefficient of x^0 in $\left(3x^2 - \frac{1}{2x^5}\right)^7$.

Ans. $\frac{7.3^6}{4}$

Sol. $T_{r+1} = {}^7C_r\,(3x^2)^{7-r}\left(-\frac{1}{2x^5}\right)^r$

$T_{r+1} = {}^7C_r\,3^{7-r}\left(-\frac{1}{2}\right)^r x^{14-7r}$.

For term independent of x : $14 - 7r = 0 \Rightarrow r = 2$

Required coefficient = $^7C_2\, 3^{7-2}\left(-\frac{1}{2}\right)^2 = \frac{7.6}{2}.3^5.\frac{1}{2^2}$

$$= \frac{7.3^6}{4}$$

11. If $(1+x)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ...a_nx^n$ and $a_r : a_{r+1} : a_{r+2} = 1 : 5 : 20$ then find a_4.

Ans. $^{29}C_4$

Sol. $a_r = {}^nC_r$

$\frac{a_r+1}{a_r} = 5 \Rightarrow \frac{^nC_{r+1}}{^nC_r} = \frac{(n+1)-(r+1)}{r+1} = 5 \Rightarrow n+1 = 6\,(r+1)$ _______(1)

$\frac{a_{r+2}}{a_{r+1}} = 4 \Rightarrow \frac{^nC_{r+2}}{^nC_{r+1}} = \frac{(n+1)-(r+2)}{r+2} = 4 \Rightarrow n+1 = 5(r+2)$ _______(2)

(1) & (2) $6\,(r+1) = 5(r+2) \Rightarrow r = 4$

$n + 1 = 30 \Rightarrow n = 29$

$a_4 = {}^{29}C_4$

12. $f(n) = \frac{n^3}{n^4+147}$ $(n \in N)$ then maximum value of f(n) is

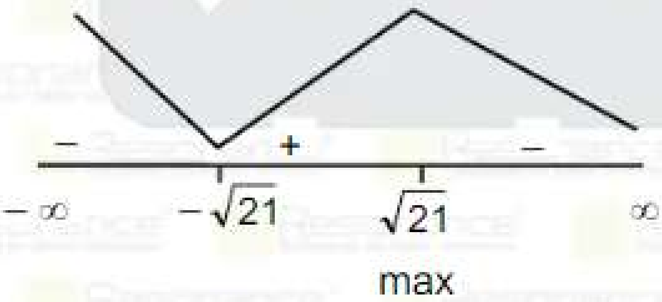
(1) $\frac{125}{772}$ (2) $\frac{64}{403}$ (3) $\frac{63}{403}$ (4) $\frac{127}{772}$

Ans. **(1)**

Sol. $f'(n) = \frac{3n^2(n^4+147)-4n^3(n^3)}{(n^4+147)^2}$

$f'(n) = \frac{n^2[3n^4+441-4n^4]}{(n^4+147)^2}$

$f'(n) = \frac{-n^2(n^4-441)}{(n^4+147)^2} = \frac{-n^2(n^2+1)(n^2-21)}{(n^4+147)^2} = \frac{-n^2(n^2+21)}{(n^4+147)^2}\,(n+\sqrt{21})(n-\sqrt{21})$



When n = 4 $a_4 = \frac{43}{4^4+147} = \frac{64}{256+147} = \frac{64}{403} =$

When n = 5 $a_5 = \frac{125}{625+147} = \frac{125}{772} = \frac{125}{772}$

a_5 is max $= \frac{125}{772}$

13. If $f(x) = \dfrac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos(x)}$ then find f (7π/4)f " (7π/4)

Ans. **(1)**

Sol. $f(x) = \sqrt{2}\dfrac{\left(\frac{1}{2}\sin x + \frac{1}{\sqrt{2}}\cos x\right) - 1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)} = \dfrac{\sin(x + \pi/4) - 1}{\sin(x - \pi/4)}$

$f'(x) = \dfrac{\cos(x + \pi/4)\sin(x - \pi/4) - \cos(x - \pi/4)(\sin(x + \pi/4) - 1)}{\sin^2(x - \pi/4)}$

$f'(x) = \dfrac{\cos(x - \pi/4) - 1}{\sin^2(x - \pi/4)} = \dfrac{-(1 - \cos(x - \pi/4))}{1 - \cos^2(x - \pi/4)}$

$f'(x) = -\dfrac{1}{1 + \cos(x - \pi/4)}$

$= f''(x) = -\dfrac{\sin(x - \pi/4)}{(1 + \cos(x - \pi/4))^2}$

$f(7\pi/4) = -\dfrac{\sin(2\pi) - 1}{\sin(3\pi/2)} = \dfrac{-1}{-1} = 1$

$f''(7\pi/4) = \dfrac{-\sin(3\pi/2)}{(1 + \cos(3\pi/2))^2} = \dfrac{1}{(1)^2} = 1$

$f(7\pi/4)\, f''(7\pi/4) = 1$

14. The shortest distance between lines

$\dfrac{x-5}{4} = \dfrac{y-3}{6} = \dfrac{z-2}{4}$ and $\dfrac{x-3}{7} = \dfrac{y-2}{5} = \dfrac{z-9}{6}$

(1) $\dfrac{59}{\sqrt{981}}$ (2) $\dfrac{95}{\sqrt{189}}$ (3) $\dfrac{53}{\sqrt{71}}$ (4) $\dfrac{73}{\sqrt{153}}$

Ans. **(2)**

Sol. $\bar{n}_1 = 4\hat{i} + 6\hat{j} + 4\hat{k}$ or $\bar{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\bar{n}_2 = 7\hat{i} + 5\hat{j} + 6\hat{k}$

$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 7 & 5 & 6 \end{vmatrix} = \hat{i}(8) + 2\hat{j} - 11\hat{k}$

A = (5, 3, 2) B = (3, 2, 9)

$\overline{AB} = -2\hat{i} - \hat{j} + 7\hat{k}$

$S.D = \dfrac{|\overline{AB}.(\bar{n}_1 \times \bar{n}_2)|}{|\bar{n}_1 \times \bar{n}_2|}$

$= \dfrac{|-16 - 2 - 77|}{\sqrt{64 + 4 + 121}} = \dfrac{95}{\sqrt{189}}$

15. If $\vec{a}$ and $\vec{b}$ are two vector such that $\vec{a}\cdot\vec{b} = 12$ and $\vec{a}\times\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ then Product of magnitude of two vector is

(1) $\sqrt{260}$ (2) $\sqrt{240}$ (3) $\sqrt{250}$ (4) $\sqrt{270}$

Ans. **(1)**

Sol. Given $\vec{a}\cdot\vec{b} = 12$, $\vec{a}\times\vec{b} = 4\hat{i}+6\hat{j}+8\hat{k}$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{4^2+6^2+8^2} = \sqrt{116}$$

We know $(\vec{a}\cdot\vec{b})^2 + (\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

$$\left|\vec{a}\right|\left|\vec{b}\right|^2 = \sqrt{(12)^2+\left(\sqrt{116}\right)^2} = \sqrt{260}$$

16. If $S_k = \dfrac{1+2+3+.....+k}{k}$ then $\sum\limits_{j=1}^{n} S_j^2 =$

(1) $\dfrac{n(n+1)(n+2)(2n+3)-6}{24}$ (2) $\dfrac{(n+1)(n+2)(2n+3)-6}{24}$

(3) $\dfrac{n(n+1)(n+2)(2n+3)-6}{6}$ (4) $\dfrac{(n+1)(n+2)(2n+3)-6}{4}$

Ans. **(2)**

Sol. $S_k = \dfrac{\frac{k(k+1)}{2}}{k} = \dfrac{1}{2}(k+1)$

$$\sum_{k=1}^{n} S_k^2 = \frac{1}{4}\sum_{k=1}^{n}(k+1)^2 = \frac{1}{4}\left(\left(2^2+3^3+......+(n+1)^2\right)+1^2-1^2\right)$$

$$= \frac{1}{4}\left(\frac{(n+1)(n+2)(2(n+1)+1)}{6}-1\right)$$

$$= \frac{1}{4}\left(\frac{(n+1)(n+2)(2n+3)-6}{6}\right)$$

$$= \frac{(n+1)(n+2)(2n+3)-6}{24}$$

17. If $f(x) = \dfrac{(1+2^x)^7}{2^x}$, $x \in R$, then which of the following is correct .

(1) $f(x)$ is even function (2) $f(x)$ is odd function

(3) $f(x)$ is neither even nor odd function (4) none

Ans. **(3)**

Sol. $f(-x) = \dfrac{(1+2^{-x})^7}{2^{-x}}$

$$= \frac{(2^x+1)^7}{2^{6x}} \neq f(x)$$

$f(-x) \neq -f(x)$

Neither even nor odd

18. In a certain factory machines A, B and C produce bolts. of their production A, B and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts machine B produces 25% and machine C produces 40%. A bolts is chosen at random from the factory's production and its found to be defective. The odds in favour that it was produced on machine C is

Ans. **01.26**

Sol. M : Bolt is defective

B_1 : Produced by A ; $P(B_1) = \frac{35}{100} = \frac{7}{20}$

B_2 : Produced by B ; $P(B_2) = \frac{25}{100} = \frac{5}{20}$

B_3 : Produced by C ; $P(B_3) = \frac{40}{100} = \frac{8}{20}$

$$P\left(\frac{B_3}{M}\right) = \frac{P(B_3)\ .\ P(M/B_3)}{\Sigma\ P(B_1)\ P(M/B_1)} = \frac{(.3)\left(\frac{8}{20}\right)}{(.2)\left(\frac{7}{20}\right)+(.1)\left(\frac{5}{20}\right)+(.3)\left(\frac{8}{20}\right)} = \frac{24}{14+5+24} = \frac{24}{43}$$

19. How many subsets of $A \times B$ are possible such that it has atleast 3 elements and not more then six elements, if A has 5 elements & B has 2 elements

Ans. **792**

Sol. $n(A) = 5, \quad n(B)=2$

$\therefore n(A\times B) = 5 \times 2 = 10$

Number of subsets = ${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

= 120+420 + 252 = 792

20. If $\left(e, \frac{4}{3}\right)$ & (e^4, α) satisfy differential equation $\frac{dy}{dx} + \frac{y}{2x\ell nx} = \frac{1}{x}$ then find α

Ans. **(3)**

Sol. $\frac{dy}{dx} + \frac{y}{2x\ell nx} = \frac{1}{x}$ linear in y

$$\text{I.F.} = e^{\int\frac{dx}{2x\ell nx}} = e^{\frac{1}{2}\ell n\ell nx} = \sqrt{\ell nx}$$

$$\therefore \text{solution is } y\sqrt{\ell nx} = \frac{(\ell nx)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\text{At } \left(e, \frac{4}{3}\right)\ \frac{4}{3}.1 = \frac{1}{\frac{3}{2}} + c \Rightarrow C = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$\therefore y\sqrt{\ell nx} = \frac{2}{3}(\ell nx)^{\frac{3}{2}} + \frac{2}{3} \qquad \therefore \text{at } (e^4 . \alpha)$$

$$\alpha = 3 \qquad \alpha.2 = \frac{2}{3}.8 + \frac{2}{3} = 6$$