44 YEARS OF EXCELLENCE

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NARAYANA GRABS
THE LION'S SHARE IN JEE-ADV.2022
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Memory Based Question Paper

MATHEMATICS

MATHEMATICS

1. The value of $\int_0^{\infty} \frac{6dx}{e^{3x}+6e^{2x}+11e^x+6}$ equal to

(1) $\ell n\left(\frac{32}{9}\right)$ (2) $\ell n\left(\frac{9}{8}\right)$ (3) $\ell n\left(\frac{8}{27}\right)$ (4) $\ell n\left(\frac{32}{27}\right)$

Ans. **(4)**

Sol. Let $e^x = t$

$e^x\,dx = dt$

$$I = \int_1^{\infty} \frac{\frac{6}{t}dt}{t^3+6t^2+11t+6} = \int_1^{\infty} \frac{6}{t(t+1)(t+2)(t+3)}\,dt$$

$$= \int_1^{\infty} \left(\frac{1}{t} - \frac{3}{t+1} + \frac{3}{t+2} - \frac{1}{t+3}\right) dt$$

$$= \left\{\ell nt - 3\ell n(t+1) + 3\ell n(t+2) - \ell n(t+3)\right\}_1^{\infty}$$

$$= \left\{\ell n\left\{\frac{t(t+2)^3}{(t+1)^3(t+3)}\right\}\right\}_1^{\infty} = \ell n\,(1) - \ell n\,\frac{27}{32} = -\,\ell n\,\frac{27}{32}$$

2. Consider the functional equation $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$ then $\left|f(3) - f'\left(\frac{1}{4}\right)\right|$ is equal to

(1) 5 (2) –3 (3) 0 (4) 7

Ans. **(4)**

Sol. $\mathbf{x \to \frac{1}{x}}$

$$3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$$

$$3f\left(\frac{1}{x}\right) + 2f(x) = x - 10$$

$$\frac{3}{2}\left[\frac{1}{x} - 10 - 3f(x)\right] + 2f(x) = x - 10$$

$$\Rightarrow \frac{3}{2x} - 15 - \frac{9}{2}f(x) + 2f(x) = x - 10 \Rightarrow \frac{-5}{2}f(x) = x - \frac{3}{2x} + 5$$

$$f(x) = -\frac{2}{5}x + \frac{3}{5x} - 2$$

$$f(3) = \frac{-6}{5} + \frac{1}{5} - 2 = -3$$

$$f'(x) = \frac{-2}{5} - \frac{3}{5x^2} \Rightarrow f'\left(\frac{1}{4}\right) = \frac{-2}{5}\,\frac{-48}{5} = -10$$

$$\left|f(3) - f'\left(\frac{1}{4}\right)\right| = |-3 + 10| = 7$$

3. Let $y_1(x)$ and $y_2(x)$ satisfy the differential equation $\frac{dy}{dx} = y + 7$. If $y_1(0) = 0$ & $y_2(0) = 1$, find the number of intersection of $y_1(x)$ & $y_2(x)$

(1) 1 (2) 0 (3) 2 (4) 3

Ans. **(2)**

Sol. $\int \frac{dy}{y+7} = \int dx \Rightarrow |y + 7| = ke^x$

$\Rightarrow y_1 + 7 = k_1e^x$ & $y_2 + 7 = k_2e^x$

$y_1(0) = 0 \Rightarrow k_1 = 7$ and $y_2(0) = 1 \Rightarrow k_2 = 8$

$\therefore\ y_1(x) = 7(e^x - 1)$ & $y_2(x) = 8e^x - 7$

Intersection $7e^x - 7 = 8e^x - 7$

$\Rightarrow e^x = 0 \Rightarrow x \in \phi$

$\therefore$ No point of intersection

4. If PQ is a focal chord of $y^2 = 36x$ with PQ = 100 and M divides PQ in the ratio 3 : 1, then the line through M perpendicular to PQ pass through -

(1) (1, 0) (2) (2, 3) (3) (3, 33) (4) (4, 4)

Ans. **(3)**

Sol. $9\left(t+\frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow P(81, 54)$ & $Q(1, -6)$

$M(21,9)$

$\Rightarrow$ L is $(y - 9) = \frac{-4}{3}(x-21)$

$3y - 27 = -4x + 84$

$4x + 3y = 111$

5. Number of symmetric matrices of order 3 × 3 which can be constructed from the elements {0, 1, 2, 3, ..., 9} is

(1) 10^5 (2) 10^9 (3) 10^4 (4) 10^6

Ans. **(4)**

Sol. $\begin{bmatrix} x & a & b \\ a & x & c \\ b & c & x \end{bmatrix}$

a can be filled in 10 ways

b can be filled in 10 ways

c can be filled in 10 ways

elements of diagonal can be filled in 10^3 ways

total no of matrices = $10^3 \times 10^3 = 10^6$

6. Let the frequency distribution is

x_i	1	3	5	7	9
f_i	4	24	28	α	8

If mean of observation is 5 then find $\frac{3\alpha}{\text{Mean deviation about mean + variance}}$

(1) 8 (2) 7 (3) 4 (4) 6

Ans. **(1)**

Sol. $5 = \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4 + 72 + 140 + 7\alpha + 72}{64 + \alpha}$

$\Rightarrow 320 + 5\alpha = 288 + 7\alpha \Rightarrow 2\alpha = 32 \Rightarrow \alpha = 16$

$\text{M.D.}(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$ where $\sum f_i = 64 + 16 = 80$

$\text{M.D.}(\bar{x}) = \frac{4\times4 + 24\times2 + 28\times0 + 16\times2 + 8\times4}{80}$

$= \frac{128}{80} = \frac{8}{5}$

$\text{variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$

$= \frac{4\times16 + 24\times4 + 0 + 16\times4 + 8\times16}{80} = \frac{352}{80}$

$\frac{3\alpha}{\text{Mean deviation about mean + variance}} = \frac{3\times16}{\frac{128}{80} + \frac{352}{80}} = \frac{3\times16\times80}{480} = \frac{3\times8}{3} = 8$

7. Let $S_1 : \lim_{n\to\infty} \frac{2 + 4 + 6.... + 2n}{n^2} = \frac{1}{2}$

$S_2 : \lim_{n\to\infty} \frac{1^{15} + 2^{15} + ... + n^{15}}{n^{16}} = \frac{1}{16}$

(1) S_1 is false, S_2 is true (2) S_1 is true, S_2 is false

(3) Both are true (4) Both are false

Ans. **(1)**

Sol. $S_1 : \lim_{n\to\infty} 2 . \frac{n(n+1)}{2 . n^2} = 1$

$S_2 : \lim_{n\to\infty} \frac{\left(\frac{n^{16}}{16} + ...\right)}{n^{16}} = \frac{1}{16}$

8. $2.2^2 - 3^2 + 2.4^2 - 5^2 + \ldots\ldots$ (upto 20 terms) is equal to

Ans. **(1310)**

Sol. $(2^2 + 4^2 + \ldots\ldots\ldots + 20^2) + (2^2 - 3^2 + 4^2 - 5^2 + \ldots\ldots\ldots + 20^2 - 21^2)$

$= 2^2(1^2 + 2^2 + \ldots\ldots\ldots + 10^2) - (2 + 3 + 4 + \ldots\ldots + 21)$

$= 4 \cdot \frac{10.11.21}{6} - \left(\frac{21.22}{2} - 1\right)$

$= 1310$

9. Let $g(x) = \sqrt{x+1}$ and $f(g(x)) = 3 - \sqrt{x+1}$, then find f(0)

(1) 3 (2) 2 (3) –1 (4) 0

Ans. **(1)**

Sol. $g(x) = 0 \Rightarrow x = -1$

$f(g(-1)) = 3 - \sqrt{-1+1} = 3$

$\Rightarrow f(0) = 3$

10. The number of seven digit numbers formed using 1, 2, 3, 4 whose sum of digits is 12, is

(1) 402 (2) 413 (3) 421 (4) 409

Ans. **(2)**

Sol. digits used (4, 3, 1, 1, 1, 1,1)

digits used (4, 2, 2, 1, 1, 1,1)

digits used (3, 3, 2, 1, 1, 1,1)

digits used (3, 2, 2, 2, 1, 1,1)

digits used (2, 2, 2, 2, 2, 1,1)

Total ways = $\frac{7!}{5!} + \frac{7!}{2!4!} + \frac{7!}{2!4!} + \frac{7!}{3!3!} + \frac{7!}{5!2!}$

$= 42 + 105 + 105 + 140 + 21 = 413$

Alter

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12$

$x_i \in \{1, 2, 3, 4\}$

coeff. of x^{12} in $(x + x^2 + x^3 + x^4)^7 = x^7(1 - x^4)^7(1 - x)^{-7}$

coeff. of x^5 in $(1 - x^4)^7(1 - x)^{-7}$

$= {}^7C_0\,{}^{11}C_5 - {}^7C_1\,{}^7C_1 = 413$

11. Let 1, 2, 3, ………………… 10 are first terms of 10 A.P.s respectively. If the common difference of these 10 AP is are 1, 3, 5, 7, ….. respectively then find $\sum_{i=1}^{10} S_i$ where S_i represents the sum of first 12 terms of i^{th} A.P.

(1) 7260 (2) 7240 (3) 7230 (4) 7220

Ans. **(1)**

Sol. $S_i = \frac{12}{2}\left[2i + 11\times(2i-1)\right] = 6\left[24i - 11\right]$

$\sum_{i=1}^{10} S_i = 6\sum_{i=1}^{10}(24i - 11) = 6\left[\frac{24\times10\times11}{2} - 110\right]$

$= 6[1320 - 110] = 1210 \times 6 = 7260$

12. Find $\left\{\frac{4^{2022}}{15}\right\}$ {.} is fraction part function

(1) $\frac{1}{15}$ (2) $\frac{4}{15}$ (3) $\frac{7}{15}$ (4) $\frac{1}{15}$

Ans. **(1)**

Sol. $4^{2022} = 16^{1011} = (15 + 1)^{1011} = 15\lambda + 1$

$\Rightarrow \left\{\frac{4^{2022}}{15}\right\} = \frac{1}{15}$

13. Consider the binomial expansion $\left(\sqrt{x} - \frac{6}{x^{3/2}}\right)^n$ where $n \le 15$. If the coefficient of term independent of x is α and sum of coefficients of all terms except the coefficient of independent term is 649 then find λ where coefficient of x^{-n} is 24λ

(1) –36 (2) –18 (3) –9 (4) –6

Ans. **(1)**

Sol. $T_{r+1} = {}^nC_r (x)^{\frac{n-r}{2}} (-6)^r x^{-3r/2}$

$= {}^nC_r (x)^{\frac{n-4r}{2}} (-6)^r$

$\frac{n-4r}{2} = 0 \Rightarrow r = \frac{n}{4}$

$x = 1 \Rightarrow (-5)^n$

So $(-5)^n - {}^nC_{n/4}(-6)^{n/4} = 649 \Rightarrow n = 4$

Now $\frac{n-4r}{2} = -n \Rightarrow 3n = 4r \Rightarrow r = 3$

so coefficient of x^{-n} is ${}^4C_3(-6)^3 = 4 \times -216 = 24\lambda$

$\lambda = -36$

14. Let $f(x) = x - \sin 2x + \frac{\sin 3x}{3}$ defined on $[0, \pi]$ then the maximum value of f(x) is.

(1) $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{3}$ (2) $\frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{2}{3}$

(3) $\frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$ (4) $\pi + \frac{\sqrt{3}}{2} + \frac{1}{3}$

Ans. **(3)**

Sol. $f'(x) = 1 - 2\cos(2x) + \cos(3x) = 0$

$\cos x = t \Rightarrow 4t^3 - 3t - 2(2t^2 - 1) + 1 = 0$

$4t^3 - 4t^2 - 3t + 3 = 0$

$4t^2(t-1) - 3(t-1) = 0 \Rightarrow t = 1,\ t = \pm\frac{\sqrt{3}}{2}$

$\cos x = 1 \Rightarrow x = 0,\ \cos x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f''(x) = 4\sin(2x) - 3\sin(3x)$

$f''\left(\frac{\pi}{6}\right) = 4\times\frac{\sqrt{3}}{2} - 3 = 2\sqrt{3} - 3 > 0$

$f''\left(\frac{5\pi}{6}\right) = 4\times\left(-\frac{\sqrt{3}}{2}\right) - 3 = -2\sqrt{3} - 3 < 0$

$f(0) = 0,\ f(\pi) = \pi,\ f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{3}$

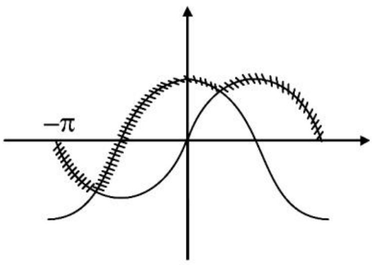
$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3} \rightarrow$ Maximum as $f''\left(\frac{5\pi}{6}\right) < 0$

15. Let $y = f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ find the area bounded by $y = f(x)$ and the x-axis

(1) 8 (2) 6 (3) 4 (4) $\frac{4}{3}$

Ans. **(3)**

Sol.



$$\int_{-\pi}^{-3\pi/4}(-\sin x)\,dx + \int_{-3\pi/4}^{-\pi/2}(-\cos x)\,dx + \int_{-\pi/2}^{\pi/4}\cos x\,dx + \int_{\pi/4}^{\pi}\sin x\,dx$$

$$= (\cos x)_{-\pi}^{-3\pi/4}dx - (\sin x)_{-3\pi/4}^{-\pi/2} + (\sin x)_{-\pi/2}^{\pi/4} - (\cos x)_{\pi/4}^{\pi}$$

$$= \left(-\frac{1}{\sqrt{2}} + 1\right) - \left(-1 + \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} + 1\right) - \left(-1 - \frac{1}{\sqrt{2}}\right)$$

$= 4$

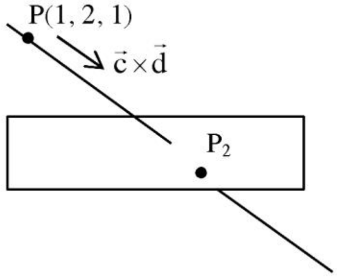
16. Find the distance of the point P(1, 2, 1) from the plane x + y + z = 6 measured along the line of shortest distance for the lines $\frac{x-1}{1} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$

(1) $4\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{2}$ (4) $4\sqrt{2}$

Ans. **(2)**

Sol. $\vec{c}\times\vec{d}$ = <4, –4, 4>

L is $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$



$P_2\ (\lambda + 1, -\lambda + 2, \lambda + 1)$

$x + y + z = 6 \Rightarrow \lambda = 2$

$\therefore \quad P_2(3, 0, 3)$

$\Rightarrow \quad PP_2 = \sqrt{4+4+4} = 2\sqrt{3}$

17. The values of α so that equation $x|x-1| + |x+2| + \alpha = 0$ have exactly one solution

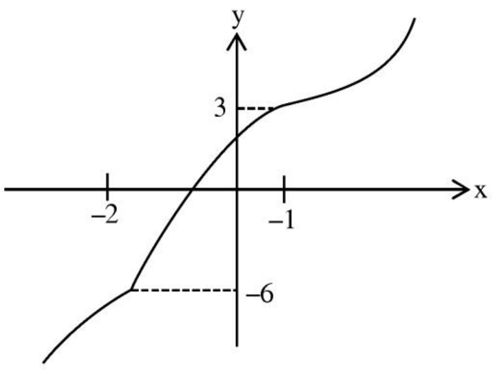
(1) $(-\infty, \infty)$ (2) $(-6, 3)$ (3) $(3, \infty)$ (4) $(-\infty, -6)$

Ans. **(1)**

Sol. $x|x-1| + |x+2| = -\alpha$

let $y = x|x-1| + |x+2|$

$$y = \begin{cases} x^2 + 2 & ; \quad x \ge 1 \\ -x^2 + 2x + 2 & ; \quad -2 \le x < 1 \\ -x^2 - 2 & ; \quad x < -2 \end{cases}$$



as $y \in R$

hence $-\alpha \in (-\infty, \infty)$

18. Let probability of getting head is thrice of probability of getting tail. A coin is tossed until one head or 3 tails are obtained. If number of trials is 24 then the mean of experiment is

(1) 28.5 (2) 31.5 (3) 25 (4) 26.5

Ans. **(2)**

Sol. **Case-I** $H \to 1 \times \frac{3}{4}$

Case-II $TH \to 2 \times \frac{1}{4} \times \frac{3}{4}$

Case-III $TTH \to 3 \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}$

Case-IV $TTT \to 3 \times \left(\frac{1}{4}\right)^3$

Mean = $\left(\frac{3}{4} + \frac{6}{16} + \frac{9}{64} + \frac{3}{64}\right) \times 24$

$= 31.5$

19. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + 7\hat{k}$, $\vec{c} = 7\hat{i} + 8\hat{j} + 9\hat{k}$. If $\vec{a} \times \vec{b} = \vec{c} + \vec{d}$ then $|\vec{d}|$ is

(1) $\sqrt{194}$ (2) $\sqrt{190}$ (3) $\sqrt{187}$ (4) $\sqrt{185}$

Ans. **(1)**

Sol. $\vec{d} = \vec{a} \times \vec{b} - \vec{c}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & 3 & 7 \end{vmatrix} = 6\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{d} = \left(6\hat{i} + \hat{j} - 3\hat{k}\right) - \left(7\hat{i} + 8\hat{j} + 9\hat{k}\right) = -\hat{i} - 7\hat{j} - 12\hat{k}$$

$$\left|\vec{d}\right| = \sqrt{1 + 49 + 144} = \sqrt{194}$$

20. If $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$ (for $x \geq 0$) then $\sin\left((x^2+x+5)\frac{\pi}{2}\right) - \cos((x^2 + x + 5)\pi)$ is

(1) 1 (2) 3 (3) 2 (4) 4

Ans. **(3)**

Sol. $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) = \tan^{-1}(x + 1)$

$\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \tan^{-1}x$

$\Rightarrow \tan^{-1}(x + 1) - \tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{1}{1+x(x+1)}\right) = \frac{\pi}{4}$

$\Rightarrow 1 + x(x + 1) = 1 \Rightarrow x = 0, x = -1$

so $\sin\left(\frac{5\pi}{2}\right) - \cos(5\pi) = 1 - (-1) = 2$

21. If $\frac{dy}{dx} = 6e^x + e^{2x} + e^{3x}$, then y(2) – y(0) is

(1) $e^2 + \frac{6e^4}{4} - \frac{e^6}{3} + \frac{15}{6}$ (2) $6e^2 + \frac{e^4}{3} + \frac{e^6}{2} - \frac{15}{6}$

(3) $6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{41}{6}$ (4) $e^2 + \frac{6e^4}{2} + \frac{e^6}{3} - \frac{15}{6}$

Ans. **(3)**

Sol. $y = 6\int e^x dx + \int e^{2x} dx + \int e^{3x} dx$

$y = 6e^x + \frac{e^{2x}}{2} + \frac{e^{3x}}{3} + c$

$y(2) = 6e^2 + \frac{e^4}{2} + \frac{e^6}{3}$, $y(0) = 6 + \frac{1}{2} + \frac{1}{3} = \frac{41}{6}$

$y(2) - y(0) = 6e^2 + \frac{e^4}{2} + \frac{e^6}{3} - \frac{41}{6}$

22. Plane P_3 is passing through the point (1, 1, 1) and line of intersection of P_1 and P_2 where $P_1 : 2x - y + z = 5$ and $P_2 : x + 3y + 2z + 2 = 0$, then distance of (1, 1, 10) from P_3 is

(1) $\frac{126}{\sqrt{558}}$ (2) $\frac{63}{\sqrt{558}}$ (3) $\frac{252}{\sqrt{558}}$ (4) None of these

Ans. **(1)**

Sol. P_3 is $P_1 + \lambda P_2 = 0$

$(2x - y + z - 5) + \lambda(x + 3y + 2z + 2) = 0$

$-3 + \lambda(8) = 0 \Rightarrow \lambda = 3/8$

$\therefore P_3 : 19x + y + 14z = 34$

distance = $\left|\frac{19+1+140-34}{\sqrt{361+1+196}}\right| = \frac{126}{\sqrt{558}}$

23. The negation of $((A \wedge (B \vee C)) \to (B \wedge C)) \to A$ is equivalent to

(1) ~B (2) ~(A ∧ B) (3) ~A (4) ~(A ∨ B)

Ans. **(3)**

Sol. $((A \wedge (B \vee C)) \to (B \wedge C)) \equiv \sim(A \wedge (B \vee C)) \vee (B \wedge C)$

So $((A \wedge (B \vee C)) \to (B \wedge C)) \to A$

$\equiv (\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \to A$

$\equiv \sim(\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \to A)$

$\equiv \sim(\sim(\sim(A \wedge (B \vee C)) \vee (B \wedge C)) \vee A)$

$\equiv \sim(A \wedge (B \vee C)) \wedge \sim(B \wedge C)) \vee A)$

$\equiv \sim A$

24. Consider the system of linear equations

$$2x + 4y + bz = 2a$$

$$x + y + z = 6$$

$$x - y - z = 8$$

then the correct option are -

(1) If a = 5 and b = 4 then system has unique solution

(2) If b ≠ 4 then system has unique solution only for a = 1

(3) If a = 5 and b = 4 then system has infinite solution

(4) If a = 5 and b = 4 then system has no solution

Ans. **(3)**

Sol. $D = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 4 & b \\ 1 & 1 & 1 \\ 2 & 0 & 0 \end{vmatrix} = 2(4-b)$

$D_1 = \begin{vmatrix} 2a & 4 & b \\ 6 & 1 & 1 \\ 8 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2a & 4 & b \\ 6 & 1 & 1 \\ 14 & 0 & 0 \end{vmatrix} = 14(4-b)$

$D_2 = \begin{vmatrix} 2 & 2a & b \\ 1 & 6 & 1 \\ 1 & 8 & -1 \end{vmatrix}$

$2[-14]-2a(-2) + b(2) = 4a + 2b - 28$

$D_3 = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 1 & 6 \\ 1 & -1 & 8 \end{vmatrix} = 2[14]-4(2) + 2a(-2)$

$= 28 - 8 - 4a = 20 - 4a$