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JEE (MAIN) 2024

MEMORY BASED QUESTIONS & SOLUTIONS

SHIFT-2

DATE & DAY: 31st January 2024 & Wednesday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM - 06:00 PM

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. The number of solutions of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) 1 (2) 0 (3) infinite (4) 2

Ans. **(2)**

Sol. Let $e^{\sin x} = t$

$\Rightarrow t - \frac{2}{t} = 2 \Rightarrow t^2 - 2t - 2 = 0$

$(t - 1)^2 = 3$

$t - 1 = \pm\sqrt{3}$

$t = 1 \pm \sqrt{3}$ but $t > 0$

so, $e^{\sin x} = 1 + \sqrt{3}$

Now, $-1 \le \sin x \le 1$

$e^{-1} \le e^{\sin x} \le e$

$e \simeq 2.72$ but $1+\sqrt{3} \simeq 2.73$

$\Rightarrow$ No. of solutions = 0

2. If $a = \sin^{-1}(\sin 5)$ and $b = \cos^{-1}(\cos 5)$ then value of $a^2 + b^2$ is

(1) $(2\pi - 5)^2$ (2) $(3\pi - 7)^2$ (3) $2(2\pi - 5)^2$ (4) $2(3\pi - 7)^2$

Ans. **(3)**

Sol. $a = \sin^{-1}(\sin 5)$, $b = \cos^{-1}(\cos 5)$

$a = 5 - 2\pi$, $b = 2\pi - 5$

$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$

$= 2(2\pi - 5)^2$

3. If 2^{nd}, 8^{th}, 44^{th} term of an non-constant arithmetic progression is same as 1^{st}, 2^{nd} & 3^{rd} term of Geometric progression respectively and first term of arithmetic progression is 1, then sum of first 20 terms of that arithmetic progression is

Ans. **(970)**

Sol. Let common difference = d

$t_2 = 1 + d$, $t_8 = 1 + 7d$, $t_{44} = 1 + 43d$

but these are in G.P.

$\Rightarrow (1 + 7d)^2 = (1 + d)(1 + 43d)$

$\Rightarrow 6d^2 - 30d = 0 \Rightarrow d = 5$ ($\because d \ne 0$)

Now, sum of first 20 terms of an AP $= \frac{20}{2}(2a + (20-1)d) = 10(2 + 95) = 970$

4. If $f : R \to (0, \infty)$ is an increasing function such that $\lim\limits_{x\to\infty}\frac{f(7x)}{f(x)} = 1$, then the value of $\lim\limits_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right]$ (where [•] denote the greatest integer function) is

(1) 1 (2) 0 (3) 2 (4) 3

Ans. **(2)**

Sol. $f(x) \le f(5x) \le f(7x)$, $\forall\, x > 0$

$1 \le \frac{f(5x)}{f(x)} \le \frac{f(7x)}{f(x)}$

As $x \to \infty$ $\frac{f(5x)}{f(x)} \to 1^{+}$

$\lim\limits_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right] = 0$

5. The area bounded by the curves $3y = (x - 4)^2$ and $y = 4x - x^2$ is

(1) 10 (2) 6 (3) 14 (4) 27

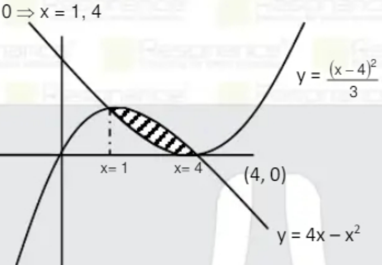
Ans. **(2)**

Sol. Solving $3y = (x - 4)^2$ and $y = 4x - x^2$

$12x - 3x^2 = x^2 - 8x + 16$

$\Rightarrow 4x^2 - 20x + 16 = 0$

$(x - 1)(x - 4) = 0 \Rightarrow x = 1, 4$



Area $= \int_1^4 \left((4x - x^2) - \frac{(x-4)^2}{3}\right)dx$

$= \left(2x^2 - \frac{x^3}{3} - \frac{(x-4)^3}{9}\right)_1^4$

$= 2(16 - 1) - \frac{1}{3}(3)(21) - \frac{1}{9}(0 + 27)$

$= 30 - 21 - 3 = 6$

6. The number of ways in which 21 identical apples to be distributed into 3 children in such a way that each children get at least 2 apples is

(1) 133 (2) 134 (3) 135 (4) 136

Ans. (4)

Sol. Let 1st student get x

2nd student get y

3rd student get z

$\Rightarrow x + y + z = 21, \quad x, y, z \geq 2$

Let $x = 2 + t_1, \quad y = 2 + t_2, z = 2 + t_3$

$\Rightarrow t_1 + t_2 + t_3 = 15$, $t_1 . t_2 . t_3 \geq 0$

number of ways = ${}^{17}C_2 = \frac{17 \times 16}{2} = 17 \times 8 = 136$

7. If $z_1 + z_2 = 5$ & $z_1^3 + z_2^3 = 20 + 15i$ then $|z_1^4 + z_2^4|$ is equal to

Ans. (75)

Sol. $z_1^3 + z_2^3 = (z_1 + z_1)((z_1 + z_2)^2 - 3z_1 z_2) \Rightarrow 20 + 15i = 5(25 - 3z_1 z_2) \Rightarrow z_1 z_2 = 7 - i$

Now $z_1^2 + z_2^2 + 2z_1 z_1 = 25 \Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i) \Rightarrow z_1^2 + z_2^2 = 11 + 2i$

Now $z_1^4 + z_2^4 + 2(7 - i)^2 = 121 - 4 + 44i \Rightarrow z_1^4 + z_2^4 = 21 + 72i$

So $|z_1^4 + z_2^4| = \sqrt{441 + (72)^2} = \sqrt{5625} = 75$

8. If A is a matrix of order 3 × 3 and det A = 2 and n = det $\underbrace{(\text{adj}(\text{adj}\ldots\ldots(\text{adj}A))}_{2024\text{times}}$, the remainder when n is divided by 9, is

(1) 2 (2) 4 (3) 6 (4) 7

Ans. (4)

Sol. $|\text{adj}A| = |A|^2$

$|\text{adj}\,\text{adj}\ldots\ldots\text{adj}(A)| = |A|^{2^{2024}} = 2^{2^{2024}}$ .... (1)

$\because 2^{2024} = 4^{1012} = (3+1)^{1012} = 3k + 1$, where k is odd

$\Rightarrow |\text{adj}(\text{adj})\ldots\ldots\text{adj}(A)| = 2^{3k+1} = 2.8^k$

$= 2(9 - 1)^k = 9m - 2$

$\Rightarrow \quad = 9P + 7$

Remainder = 7

9. A biased coin in which probability of getting head is twice to that of tail. If coin is tossed 3 times then the probability of getting two tails and one head is

(1) 1/9 (2) 2/9 (3) 2/27 (4) 1/27

Ans. (2)

Sol. $P(H) = p \quad P(T) = q \quad \Rightarrow p = 2q$ and

$p + q = 1 \Rightarrow \quad q = 1/3 \quad \Rightarrow p = 2/3$

$P(2T, 1H) = {}^3C_2\, q^2\, p$

$= {}^3C_2 . \frac{1}{9} . \frac{2}{3} = \frac{2}{9}$

10. The value of $\frac{120}{\pi^3}\left|\int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx\right|$ is

Ans. (15)

Sol. Let $I = \int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ....(1)

$I = \int_0^{\pi} -\frac{(\pi - x)^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$, by king property ....(2)

adding equation (1) and (2)

$2I = \int_0^{\pi} \frac{\pi(2x - \pi)\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$I = \pi \int_0^{\pi/2} \frac{(2x - \pi)\sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ....(3)

$I = \pi \int_0^{\pi/2} \frac{-2x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ....(4)

adding equation (3) and (4)

$$2I = -\pi^2 \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$I = -\frac{\pi^2}{2\times 2} \int_0^{\pi/2} \frac{2\sin x \cos x dx}{\sin^4 x + \cos^4 x}$$

$$I = -\frac{\pi^2}{4} \int_0^{\pi/2} \frac{2\tan x \sec^2 x}{\tan^4 x + 1} dx$$

$$I = -\frac{\pi^2}{2\times 2} \tan^{-1}\left(\tan^2 x\right)_0^{\pi/2}$$

$$I = -\frac{\pi^3}{8}$$

Now $\frac{120}{\pi^3}\left|\int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 + \cos^4 x} dx\right|$

$= \frac{120}{\pi^3} \times \left(\frac{\pi^3}{8}\right) = 15$

11. If the mean and variance of 6 observations a, b, 68, 44, 48, 60 are 55 and 194 respectively and a > b then a + 3b is

(1) 190 (2) 180 (3) 200 (4) 210

Ans. **(2)**

Sol. Mean = $\frac{a+b+68+44+48+60}{6} = 55 \Rightarrow a + b = 110$ .... (1)

Variance = $\frac{\sum(x_i - \bar{x})^2}{n} = \frac{(55-a)^2 + (55-b)^2 + (13)^2 + (11)^2 + 7^2 + 5^2}{6} = 194$

$\Rightarrow a^2 + b^2 - 110(a + b) = -5250 \Rightarrow a^2 + b^2 = 6850.$ ...... (2)

after solving equation (1) and (2), we get a = 75 & b = 35 $\Rightarrow$ a + 3b = 75 + 105 = 180

12. If $\lim_{x\to 0} \frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$ then the value of $16(a^2 + b^2 + c^2)$ is

Ans. **(81)**

Sol. $$\lim_{x\to 0} \frac{ax^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right) - b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-.....\right) + cx\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+.....\right)}{x^3\left(\frac{\sin x}{x}\right)} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{(c-b)x + \left(\frac{b}{2}-c+a\right)x^2 + \left(a-\frac{b}{3}+\frac{c}{2}\right)x^3.....}{x^3} = 1$$

$\Rightarrow c - b = 0, \quad \frac{b}{2} - c + a = 0$

and $a - \frac{b}{3} + \frac{c}{2} = 1$

$a = \frac{3}{4}, \qquad b = c = \frac{3}{2}$

$a^2 + b^2 + c^2 = \left(\frac{9}{16} + \frac{9}{4} + \frac{9}{4}\right)$

$16(a^2 + b^2 + c^2) = 9 + 36 + 36 = 81$

13. A line of negative slope passing through the centre of circle $x^2 + y^2 - 16x - 4y = 0$ intersects +ve x and y-axis at A and B respectively then the minimum value of OA + OB (O is origin) is

Ans. (18)

Sol. Let slope of line is – m

$\therefore$ Equation of straight line $(y - 2) = -m(x - 8) \Rightarrow mx + y = 8m + 2$

$A \equiv \left(8 + \frac{2}{m}, 0\right)$ $B \equiv (0, 2 + 8m)$

$OA + OB = 10 + \frac{2}{m} + 8m \geq 18$

$(OA + OB)_{min} = 18$

14. If reflection of A(1, –10, 1) about the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-2}{5}$ is (α, β, γ) then the value of $|2\alpha + 3\beta + 5\gamma|$ is.

Ans. (23)

Sol.

Let co-ordinate of foot is

$B(2\lambda + 1, 3\lambda + 1, 5\lambda + 2)$

Direction ratio of AB is $2\lambda, 3\lambda + 11, 5\lambda + 1$

$\overrightarrow{AB}$ is perpendicular to the given line so $\Rightarrow 2\lambda(2) + (3\lambda+11)3 + (5\lambda+1)5 = 0 \Rightarrow \lambda = -1$

So, foot is B(–1, –2, –3)

Now image of A (1, –10, 1) about the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-2}{5}$ is (α, β, γ)

so, $\frac{\alpha+1}{2} = -1, \frac{\beta-10}{2} = -2, \frac{\gamma+1}{2} = -3 \Rightarrow \alpha = -3, \beta = 6, \gamma = -7$

$\Rightarrow |2\alpha + 3\beta + 5\gamma| = |-6 + 18 - 35| = 23$

15. If ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} = {}^8C_3$ (where $m \neq 1$) & $\frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{8}$, then value of ${}^{n+1}C_m + {}^nP_m$ is

Ans. (420)

Sol. ${}^6C_m + {}^6C_{m+1} + {}^6C_{m+1} + {}^6C_{m+2} = {}^8C_3$

$\Rightarrow {}^7C_{m+1} + {}^7C_{m+2} = {}^8C_3$

$\Rightarrow {}^8C_{m+2} = {}^8C_3 = {}^8C_5$

$\Rightarrow m + 2 = 5$ ($\because m \neq 1$)

$\Rightarrow m = 3$

$\frac{{}^{(n-1)}P_3}{{}^nP_4} = \frac{1}{8} \Rightarrow \frac{1}{n} = \frac{1}{8} \Rightarrow n = 8$

Now ${}^{n+1}C_m + {}^nP_m = {}^9C_3 + {}^8P_3 = 84 + 336 = 420$

16. If $\frac{dT}{dt} = -k(T - 85)$ and T = 160 at t = 0 then the value of T at t = 45, is

(1) $85 + 75e^{45k}$ (2) $85 + 75e^{-45k}$ (3) $75 + 85e^{45k}$ (4) $75 + 85e^{-45k}$

Ans. (2)

Sol. $\int \frac{dt}{T - 85} = -\int k\,dt$

$\ln|T - 85| = -kt + c$

at t = 0, T = 160

$\ln 75 = c$

$\therefore |T - 85|$

$\ln\left|\dfrac{T-85}{75}\right| = -kt$

$\dfrac{T-85}{75} = +e^{-kt}$ (–rejected because T = 160 at t = 0)

$T = 85 + 75e^{-kt}$

at t = 45, $T = 85 + 75e^{-45k}$

17. If $f(x) = e^{-|\ln x|}$; $x \in (0, \infty)$ is discontinuous at m points and non-differentiable at n points then the value of m + n is

Ans. (1)

Sol. Since $|\ln x|$ is continuous in $(0, \infty)$

$\Rightarrow f(x) = e^{-|\ln x|}$ is continuous is $(0, \infty)$

$\Rightarrow$ So number of points where f(x) is discontinuous, m = 0

$$f(x) = \begin{cases} e^{\ln x}; & 0 < x < 1 \\ e^{-\ln x}; & x \ge 1 \end{cases}$$

$f'(1^-) = 1$, $f'(1^+) = -1 \Rightarrow$ So number of points where f(x) is non-differentiable, n = 1

m + n = 0 + 1 = 1

18. A is a square matrix of order 3 and v_1, v_2, v_3 are 3 column matrices such that

$$Av_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, Av_2 = \begin{bmatrix}-1\\0\\2\end{bmatrix} Av_3 = \begin{bmatrix}0\\-1\\2\end{bmatrix} \text{ where } v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, v_2 = \begin{bmatrix}2\\0\\3\end{bmatrix}, v_3 = \begin{bmatrix}1\\1\\-1\end{bmatrix} \text{ then the value of } |A| \text{ is}$$

(1) 9 (2) $\frac{9}{2}$ (3) $\frac{9}{4}$ (4) $\frac{9}{8}$

Ans. (3)

Sol. $A\begin{bmatrix}1 & 2 & 1\\1 & 0 & 1\\1 & 3 & -1\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\2 & 0 & -1\\3 & 2 & 2\end{bmatrix}$

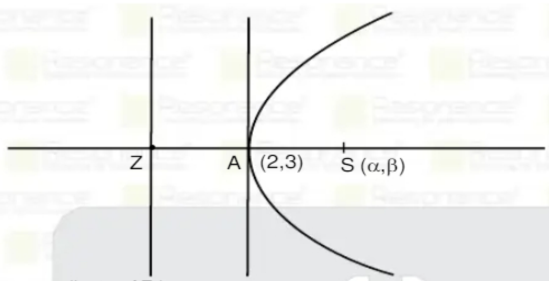
$|A|\begin{vmatrix}1 & 2 & 1\\1 & 0 & 1\\1 & 3 & -1\end{vmatrix} = \begin{vmatrix}1 & -1 & 0\\2 & 0 & -1\\3 & 2 & 2\end{vmatrix} \Rightarrow |A| = \frac{9}{4}$

19. An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) whose eccentricity is $\frac{1}{\sqrt{2}}$ and passes through the focus of the parabola whose vertex is (2,3) and directrix is 2x + y – 6 = 0 then the length of the latus rectum of ellipse is

(1) $\sqrt{656}$ (2) $\frac{\sqrt{656}}{5}$ (3) $\frac{\sqrt{656}}{10}$ (4) $\frac{\sqrt{656}}{2}$

Ans. (2)

Sol. Let foot of perpendicular from vertex of parabola on the directix is point Z



So, coordinate of Z is

$\frac{x-2}{2} = \frac{y-3}{1} = -\frac{(4+3)-6}{4+1} \Rightarrow z \equiv \left(\frac{8}{5}, \frac{14}{5}\right)$

$\Rightarrow$ focus $S \equiv \left(\frac{12}{5}, \frac{16}{5}\right)$

Now eccentricity equal to $\frac{1}{\sqrt{2}} \Rightarrow b^2 = \frac{a^2}{2}$

$\therefore \frac{144}{25a^2} + \frac{256}{25 \times \frac{a^2}{2}} = 1$

$a^2 = \frac{656}{25}$, $b^2 = \frac{328}{25}$

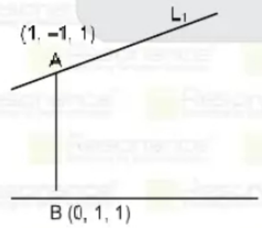
Now length of latus rectum = $\frac{2b^2}{a} = \frac{\sqrt{656}}{5}$

20. The shortest distance between the line $L_1 = (\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 14\hat{j} + 5\hat{k})$ and $L_2 = (\hat{j} + \hat{k}) + \mu(-2\hat{i} - 4\hat{j} + 7\hat{k})$ then L_1 and L_2 is

(1) $\frac{5}{\sqrt{221}}$ (2) $\frac{10}{\sqrt{221}}$ (3) $\frac{2}{\sqrt{221}}$ (4) $\frac{5}{11}$

Ans. **(1)**

Sol.



Let $\vec{p} = 2\hat{i} - 14\hat{j} + 5\hat{k}$

and $\vec{q} = -2\hat{i} - 4\hat{j} + 7\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 2 & -14 & 5 \\ -2 & -4 & 7 \end{vmatrix}$$

$= \hat{i}(-98 + 20) - \hat{j}(14 + 10) + \hat{k}(-8 - 28)$

$= -78\hat{i} - 24\hat{j} - 36\hat{k}$

$\overrightarrow{AB} = -\hat{i} + 2\hat{j}$

S.D. = $\frac{|\overrightarrow{AB}.(\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|78 - 48|}{\sqrt{(78)^2 + (24)^2 + (36)^2}} = \frac{30}{\sqrt{7956}}$

$= \frac{30}{6\sqrt{221}} = \frac{5}{\sqrt{221}}$

21. If $F : (-\infty, -1] \rightarrow (a, b]$ is defined as $f(x) = e^{x^3 - 3x + 1}$ such that F is both one–one and onto then the distance from a point $P(2a + 4, b + 2)$ to curve $x + ye^{-3} - 4 = 0$ is

(1) $\sqrt{e^3 + 2}$ (2) $\frac{e^3 + 2}{\sqrt{e^3 + 1}}$ (3) $\frac{e^3 + 2}{\sqrt{e^6 + 1}}$ (4) e

Ans. **(3)**

Sol. $f'(x) = e^{(x^3 - 3x + 1)}.3(x - 1)(x + 1)$

$a = \lim_{x \to -\infty} e^{x^3 - 3x + 1} = 0$

$b = f(-1) = e^{-1+3+1} = e^3$

$P(2a + 4, b + 2) \equiv (4, 2 + e^3)$

Distance = $\frac{|4 + (2 + e^3)e^{-3} - 4|}{\sqrt{1 + e^{-6}}} = \frac{e^3 + 2}{\sqrt{1 + e^6}}$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